Wireless Pers Commun
DOI 10.1007/s11277-012-0636-0

The Downlink Capacity of Single-User SA-MIMO System

Jianxin Dai · Ming Chen · Pei-Jung Chung

Abstract In this paper, a novel multiple antenna system framework, which combines smart antennas (SA) with multiple-input-multiple-output (MIMO) at the transmitter, is proposed. The downlink capacity of the single-user SA-MIMO wireless systems is investigated. The joint optimization problem corresponding to the capacity is deduced. After that, upper bounds of the capacity are given in general case and in the case of equal power allocation, respectively. Furthermore, in the case of equal power allocation and the same direction of departure from one transmit smart antenna to all antenna arrays at the receiver the closed-form expression of the capacity is obtained. Some numerical results are given to show that smart antennas can bring significant capacity gain for the MIMO systems due to the smart antennas gain, without additional spatial degrees of freedom, especially at high SNR with strong correlation among the MIMO channel links or at low SNR.

Keywords Smart antenna · MIMO · Capacity · Downlink · Beam-forming

1 Introduction

To provide the higher data rate, wider coverage and better quality of service (QoS) for future wireless communication, many new frameworks of multiple antenna systems have been suggested to extend conventional multiple-input-multiple-output (MIMO) systems [1–5]. The distributed antenna system (DAS) was investigated in [1,2] whose basic idea is that the

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Published online: 26 April 2012

distributed antennas were geographically separated from each other. The outage capacity of the DAS with antenna cooperation was analyzed over the Rayleigh-Log-normal fading channel in [1]. Considering the influence of path loss, lognormal shadowing and Rayleigh fading to the actual wireless environments, the downlink system capacity of DAS was studied with selective diversity scheme in [2]. The design and performance evaluation of a 3-D sum-of-sinusoids statistical simulation model were dealt with for High altitude platforms (HAP)-MIMO Ricean fading channels by applying the MIMO techniques to HAP-based communication system in [3]. In [4] the downlink performance of single cell and multi-cell MIMO relay networks were analyzed which took into account MIMO technology in fixed relay networks. Cooperative MIMO, the basic idea of which was to group multiple devices into virtual antenna arrays to emulate MIMO communications, was provided a brief survey in [5].

However, the improvement of performance in the above-mentioned multiple antenna systems comes at the price of increased cost, space and computational complexity due to the individual power constraint of each antenna, cooperation algorithm and hardware limitations. In this paper, a new smart antennas-MIMO (SA-MIMO) system with the total power constraint of all antennas is studied. The basic idea of SA-MIMO is to replace each antenna of conventional MIMO with a smart antenna array. This design is motivated by two main facts. First, MIMO system with multiple antennas at both the transmitter and receiver can increase the capacity of wireless channel significantly without requiring additional power or bandwidth [6–8], and smart antennas can suppress the interference coming from different directions by beam-forming and hence can make the cell have a wider coverage and greater user capacity [9,10]. Therefore, if combine MIMO with smart antennas, both advantages can be obtained via jointly optimizing power allocation and beam-forming vectors. Second, smart antennas are considered as a transmission technology of the single channel current in the 3rd Generation of Mobile Communication Systems (3G) standard, and MIMO technology provides multiple independent transmission channels that can increase system throughput in long term evolution (LTE) standard. Thus, it is of great interest to investigate the combination of MIMO and smart antennas in order to ensure the future system a smooth evolution, make the most of existing system resource, avoid extensive redesign of antenna and feeder system as much as possible, and reduce difficulties of network arrangement and cell-site selection. However, the investigation of the SA-MIMO is rare in the current literature.

The main contributions and novelties of this paper are listed as follows.

(1) We propose an interesting SA-MIMO system by combining the MIMO and smart antennas techniques, which is different from the traditional MIMO system. For the single-user downlink SA-MIMO system, we first propose the optimization model. This model concentrates on the joint optimizing power allocation and beam-forming vectors, which is different from conventional MIMO that only focuses on the optimization of power allocation.

(2) Compared to the conventional MIMO, the proposed SA-MIMO brings significant capacity gain without additional spatial degrees of freedom. And SA-MIMO can further suppress the interference at low SNR and reduce the channel correlation at high SNR with strong correlation among the MIMO channel links.

(3) Upper bounds of the SA-MIMO capacity, which are observed to be tight for the system studied, are given in general case and in the case of equal power allocation, respectively. And the closed-form expression of the capacity is derived in the special case of equal power allocation and the same direction of departure (DODs). The above theoretical results are useful for evaluating the capacity of the SA-MIMO system.

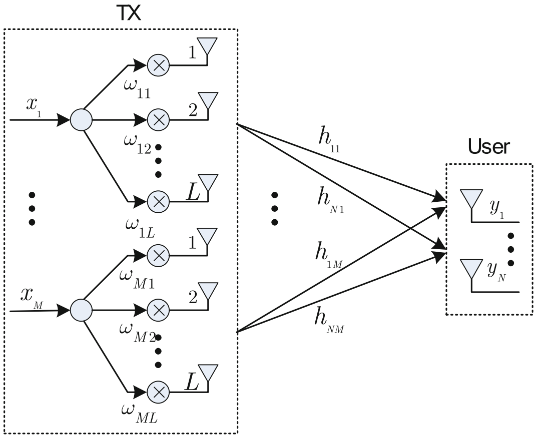

Fig. 1 Single-user downlink SA-MIMO system model

The paper is organized as follows. In Sect. 2, we introduce the single-user downlink SA-MIMO system model. Section 3 formulates the optimization problem, then upper bounds of the SA-MIMO capacity are given and the closed-form expression of the capacity is derived in the relaxation condition. Section 4 shows some numerical results. Finally, we make some concluding remarks in Sect. 5.

2 System Model

Consider a downlink SA-MIMO system which is shown in Fig. 1. At the transmitter base station, there are M antenna arrays, each of which has L elements whose mutual distances are less than the half wavelength of the carrier wave. In addition, all the M antenna arrays are so far mutually that their transmitter channels are independent of each other. The user receiver has N antennas, whose mutual distances are larger than the half wavelength of the carrier wave such that all the transmit channels are independent of each other.

Let h_{nm} be the microscope fading coefficient between the m-th transmit antenna array and the n-th receive antenna for all $1 \le m \le M$ and all $1 \le n \le N$. The channels are flat Rayleigh fading, i.e., $h_{nm} \sim CN(0, 1)$, where $x \sim CN(\mu, \sigma^2)$ means that x is complex Gaussian distributed with mean μ and variance σ^2.

Denote the beam-forming vector of the m-th array at the transmitter as $\omega_m = (\omega_{m,1}, \omega_{m,2}, \ldots, \omega_{m,L})$ with $||\omega_m|| = 1$, where $|| \cdot ||$ is the Euclidean norm of a vector. Let $\mathbf{a}_{nm} = \mathbf{a}(\varphi_{nm})$ be the steering vector of the m-th antenna array with respect to the n-th receive antenna, where φ_{nm} is the angle of incidence on the n-th receive antenna with respect to the m-th antenna array. $x_1, x_2, \ldots, x_N$ and $p_1, p_2, \ldots, p_N$ represent the transmitted signals and powers at the M antenna arrays, respectively. Then the received signal at the$n$-th antenna can be written as

$$y_n = \sum_{m=1}^{M} x_m \sqrt{p_m} h_{nm} \langle \omega_m, \mathbf{a}_{nm} \rangle + \mathbf{z}_n, \tag{1}$$

where $\mathbf{z}_n \sim CN\left(0, \sigma^2\right)$ is the noise at the n-th receive antenna, $\langle , \rangle$ denotes the Euclid inner product of two vectors.

By stacking the received signals of all antennas into $\mathbf{y} = (y_1, \ldots, y_N)^{\mathrm{T}}$, we have

$$\mathbf{y} = \mathbf{HPx} + \mathbf{z}, \tag{2}$$

where $\mathbf{H} = (h_{nm}\langle\omega_m, \mathbf{a}_{nm}\rangle)_{N\times M}$, $\mathbf{P} = \mathrm{diag}\left[\sqrt{p_1}, \ldots, \sqrt{p_M}\right]$, $\mathbf{x} = (x_1, \ldots, x_M)^{\mathrm{T}}$, $\mathbf{z} = (z_1, \ldots, z_N)^{\mathrm{T}}$.

3 Information Theoretic Capacity

The capacity is defined as the maximum of the mutual information between input and output [11], and the mutual information is

$$\mathbf{I}(\mathbf{y}; \mathbf{x}) = \mathcal{H}(\mathbf{y}) - \mathcal{H}(\mathbf{y}\,|\mathbf{x})$$

When the channel input is given, the remaining uncertainty in the output is simply the entropy of the noise. Thus we can write

$$\mathbf{I}(\mathbf{y}; \mathbf{x}) = \mathcal{H}(\mathbf{y}) - \mathcal{H}(\mathbf{z})$$

As the noise components are independent complex Gaussian variables, the entropy of $\mathbf{z}$ is $\mathcal{H}(\mathbf{z}) = \log_2(\det(\pi e\sigma^2\mathbf{I}_N))$. Therefore, the capacity is achieved if the entropy of the channel output is maximized. On the other hand, for the entropy of the channel output, we can write

$$\mathcal{H}(\mathbf{y}) \le \log_2(\det(\pi e\mathbf{R}_{\mathbf{y}}))$$

where $\mathbf{R}_{\mathbf{y}}$ denotes the covariance matrix of the output vector $\mathbf{y}$, and we have equality if $\mathbf{y}$ is circularly symmetric complex Gaussian. Clearly, the channel output vector $\mathbf{y}$ is circularly symmetric complex Gaussian if the channel input vector $\mathbf{x}$ is circularly symmetric complex Gaussian [6].

Let $\mathbb{W} = \{\omega_1, \omega_2, \ldots, \omega_M\}$. Noting that

$$\mathbf{R}_{\mathbf{y}} = \mathbb{E}[\mathbf{y}\mathbf{y}^{\mathrm{H}}] = \mathbf{HPP}^{\mathrm{H}}\mathbf{H}^{\mathrm{H}} + \sigma^2\mathbf{I}_N,$$

if the signal vector satisfies $\mathbb{E}\{\mathbf{x}\mathbf{x}^{\mathrm{H}}\} = \mathbf{I}_M$, the instantaneous downlink capacity of the SA-MIMO system for each channel user can be formulated as:

$$\begin{aligned} C = \max_{\mathbb{W},\mathbf{P}} & \left\{\log_2\det\left(\mathbf{I}_N + \frac{1}{\sigma^2}\mathbf{HPP}^{\mathrm{H}}\mathbf{H}^{\mathrm{H}}\right)\right\} \\ & \text{s.t.} \\ & \|\omega_m\| = 1, \quad \forall\omega_m \in \mathbb{W} \\ & \mathrm{Tr}\left(\mathbf{PP}^{\mathrm{H}}\right) = \sum_{m=1}^{M} p_m = P \\ & \mathbf{H} = (h_{nm}\langle\omega_m, \mathbf{a}_{nm}\rangle)_{1\le n\le N, 1\le m\le M}\ , \end{aligned} \tag{3}$$

where the superscript H denotes the complex-conjugate transpose of a vector or matrix.

denote $\rho = P/\sigma^2$, and let $\lambda_i (i = 1, 2, \ldots, N)$be the eigenvalues of the matrix $\mathbf{HH}^{\mathrm{H}}$. Using matrix decomposition, we have

$$\begin{aligned} \mathbf{H} &= (h_{nm}\langle\omega_m, \mathbf{a}_{nm}\rangle)_{1\le n\le N, 1\le m\le M} = \left(\omega_m^{\mathrm{H}}\mathbf{a}_{nm}h_{nm}\right)_{1\le n\le N, 1\le m\le M} \\ &= \begin{bmatrix} \mathbf{a}_{11}^T h_{11} & \ldots & \mathbf{a}_{1M}^T h_{1M} \\ \vdots & \ddots & \vdots \\ \mathbf{a}_{N1}^T h_{N1} & \ldots & \mathbf{a}_{NM}^T h_{NM} \end{bmatrix}_{N\times ML} \begin{bmatrix} \omega_1^* & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \omega_M^* \end{bmatrix}_{ML\times M} = \tilde{\mathbf{H}}_{N\times ML}\mathbf{W}_{ML\times M}\ , \end{aligned} \tag{4}$$

where $\tilde{\mathbf{H}}_{N\times ML}$ and $\mathbf{W}_{ML\times M}$ are the block matrix, $\mathbf{W}_{ML\times M}$ is a quasi-diagonal matrix, the superscripts T and $*$ denote the transpose and the complex-conjugate of a vector or matrix, respectively. Let $\tilde{\lambda}_i (i = 1, 2, \ldots, ML)$ be the eigenvalues of the matrix $\left(\tilde{\mathbf{H}}^{\mathrm{H}}\tilde{\mathbf{H}}\right)_{ML\times ML}$.

After substitution, the capacity can be written as

$$C = \max_{\mathbb{W},\mathbf{P}} \left\{ \log_2 \det \left(\mathbf{I}_N + \frac{1}{\sigma^2} \mathbf{H}\tilde{\mathbf{W}}\mathbf{P}\mathbf{P}^{\mathrm{H}}\mathbf{W}^{\mathrm{H}}\tilde{\mathbf{H}}^{\mathrm{H}} \right) \right\} \quad . \tag{5}$$

Then we have following theorems.

Theorem 1 *The upper bound of the downlink capacity of the SA-MIMO system can be written as*

$$C \le \sum_{i=1}^{ML} \left[\log_2 \left(\mu \tilde{\lambda}_i \right) \right]^+ ,$$

where x^+ *is defined as* $\max(x, 0)$ *and* μ *is the waterfilling level which is chosen to satisfy*

$$\sum_{i=1,\tilde{\lambda}_i \neq 0}^{ML} \left(\mu - \tilde{\lambda}_i^{-1} \right)^+ = \rho$$

The proof is given in "Appendix A".

Let $\mathbf{A}_j = \sum_{i=1}^{M} \left| h_{ij} \right|^2 \mathbf{a}_{ij}\mathbf{a}_{ij}^{\mathrm{H}}$, and $\lambda_{\max}(\mathbf{A}_j)$ be the maximum eigenvalue of matrix $\mathbf{A}_j$.

Theorem 2 *In the case that the power is allocated to the transmit antennas equally, the upper bound of the capacity of the SA-MIMO system can be written as*

$$C = \log_2 \prod_{i=1}^{R} \left(1 + \frac{\rho}{M} \lambda_i \right) \le R \log_2 \left(1 + \frac{\rho}{RM} \sum_{j=1}^{N} \lambda_{\max}(\mathbf{A}_j) \right), \tag{6}$$

where R *is the number of non-zero eigenvalues of matrix* $\mathbf{H}\mathbf{H}^{\mathrm{H}}$.

The proof is provided in "Appendix B".

Let $\bar{\mathbf{H}}_{N\times M} = \begin{bmatrix} h_{11} & \ldots & h_{1M} \\ \vdots & \ddots & \vdots \\ h_{N1} & \ldots & h_{NM} \end{bmatrix}$, $\bar{\lambda}_1, \bar{\lambda}_2, \ldots, \bar{\lambda}_M$ be the eigenvalues of the matrix $\bar{\mathbf{H}}^{\mathrm{H}}\bar{\mathbf{H}}$.

Theorem 3 *In the case that the DODs from one transmit smart antenna to all antenna arrays at the receiver are same, i.e.* $\mathbf{a}_{1m} = \mathbf{a}_{2m} = \cdots = \mathbf{a}_{Nm} \triangleq \mathbf{a}_m$, *and the power is allocated to the transmit antennas equally, the capacity of the SA-MIMO system can be formulated as:*

$$C = \sum_{i=1}^{M} \log_2 \left(1 + \frac{\rho L}{M} \bar{\lambda}_i \right), \tag{7}$$

if and only if the optimal beam-forming vector can be written as

$$\omega_i = \frac{1}{\sqrt{L}} \mathbf{a}_i, i = 1, 2, \ldots, M. \tag{8}$$

The proof is given in "Appendix C".

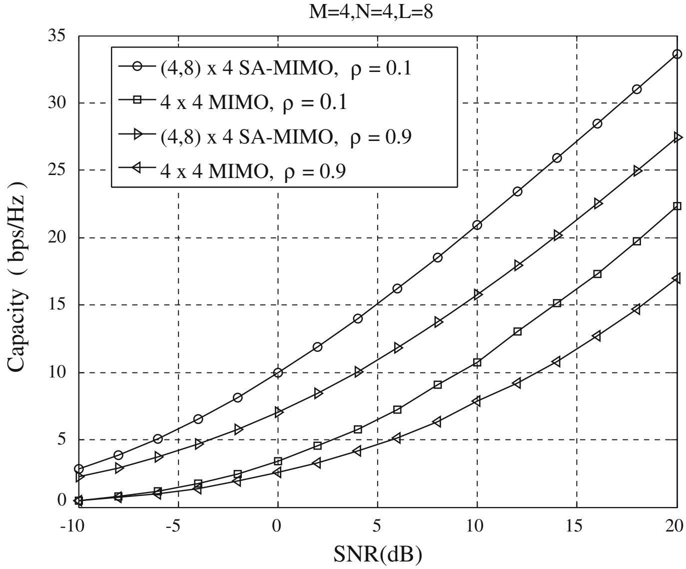

Fig. 2 Capacity comparison between MIMO and SA-MIMO versus SNR for $\rho = 0.1$, $\rho = 0.9$, respectively

4 Numerical Results

Monte Carlo simulations are carried out under some cases to compare the capacities of SA-MIMO and conventional MIMO systems. We denote $(M, L) \times N$ SA-MIMO as a SA-MIMO system with M transmit antenna arrays, each of which has L elements, and N receive antennas. In addition, denote $R = \frac{C_{\text{SA-MIMO}}}{C_{\text{MIMO}}}$ as capacity ratio of SA-MIMO to conventional MIMO, which indicates the relative capacity gain. The following simulations are performed under the assumption that the channel input vector $\mathbf{x}$ is circularly symmetric complex Gaussian with $\mathbb{E}\{\mathbf{x}\mathbf{x}^{\mathrm{H}}\} = \mathbf{I}_M$.

Figures 2 and 3 depict the capacity and capacity ratios versus SNR, respectively, with $M = 4$, $N = 4$ and $L = 8$ for weak correlation channel ($\rho = 0.1$) and strong correlation channel ($\rho = 0.9$). Figures 4 and 5 illustrate the capacity and capacity ratios versus channel correlation coefficient ρ, respectively, with $M = 4$, $N = 4$ and $L = 8$ for SNR $= -10$, 0 and 10 dB. Obviously, it can be found from Figs. 2, 3 and 4 that the capacity gain increases as the SNR increases, while the relative capacity gain improves as the SNR decreases. This is because smart antennas can suppress the interference at low SNR more easily. Additionally, relative capacity gain in the high SNR regime in Fig. 3 for strong correlation channel is higher than that for weak correlation channel. And in Fig. 5, relative capacity gain for weak correlation channel decreases as SNR increases, but that for strong correlation channel increases gradually as SNR increases. Namely smart antennas can bring relative capacity gain in the high SNR regime more significantly in the case of strong correlation than in the case of weak correlation. This is due to the reason that smart antennas can reduce channel correlation at high SNR with strong correlation among the MIMO channel links, resulting in diversity gain increases. Therefore, it can be concluded that smart antennas can bring significant capacity gain for the MIMO systems without additional spatial degrees of freedom, especially at high SNR with strong correlation among the MIMO channel links or at low SNR due to the smart antennas gain.

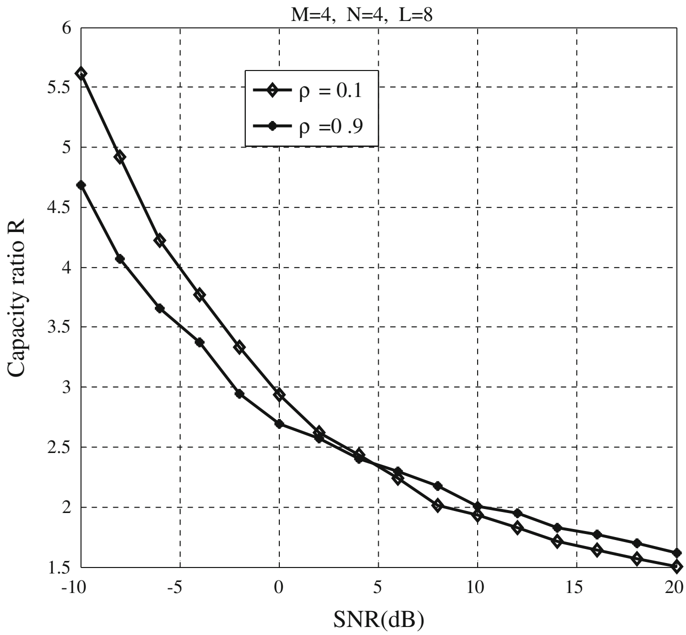

Fig. 3 Capacity ratio Comparison between MIMO and SA-MIMO versus SNR for $\rho = 0.1, \rho = 0.9$, respectively

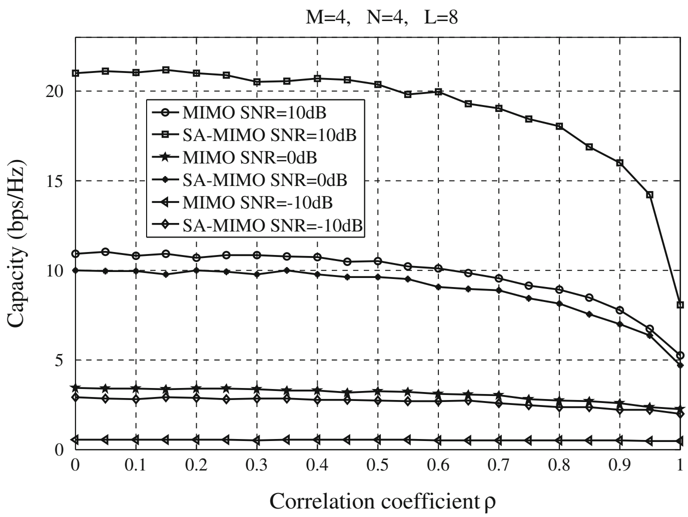

Fig. 4 Capacity Comparison between MIMO and SA-MIMO versus correlation coefficient for SNR $= -10$, 0 and 10 dB, respectively

Figure 6 depicts the ergodic channel capacity (the mutual information averaged over channel realizations) of MIMO and SA-MIMO for various values of SNR with $M = 2$, $N = 2$, $L = 4$ and $M = 4$, $N = 4$, $L = 4$, respectively. Here, the simulations are performed under the assumption that the DODs are the same at the receive antenna arrays and the channel is

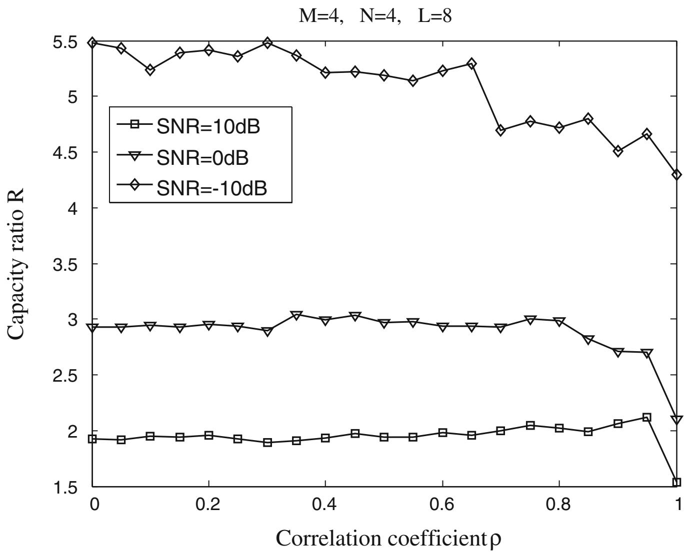

Fig. 5 Capacity ratio Comparison between MIMO and SA-MIMO versus correlation coefficient for SNR = −10, 0 and 10 dB, respectively

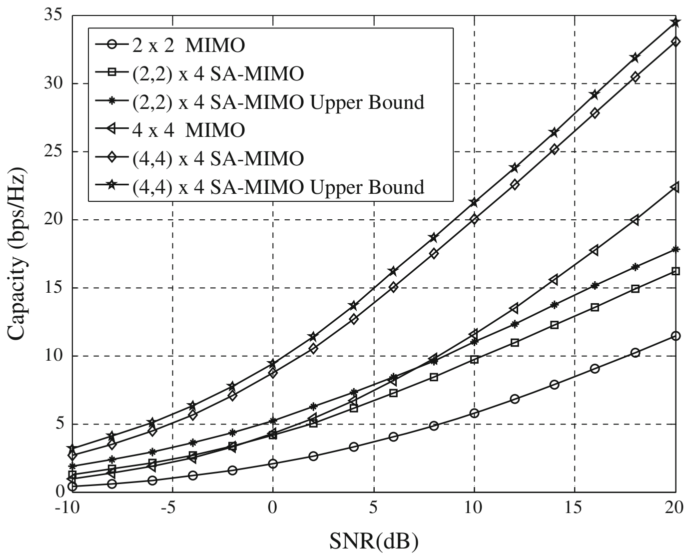

Fig. 6 Ergodic Capacity Comparison between MIMO and SA-MIMO

Rayleigh fading. It can be seen from the figure that the upper bound is actually very tight for the system considered.

5 Conclusions

This letter investigated the downlink capacity of the SA-MIMO systems. Firstly, upper bounds of the downlink capacity were derived in the general case and in the case of equal power allocation, respectively. Then, a closed-form expression of the downlink capacity was deduced in the case of the same DODs from one transmit smart antenna to all antenna arrays at the receiver with equal power allocation. Smart antennas can bring significant capacity gain for the MIMO systems without additional spatial degrees of freedom when the channel is bad-conditioned.

Forthcoming work should be focused on analyzing the capacity gain of SA-MIMO systems under the multi-user scenario in which the analysis will be much more complicated.

Acknowledgments This work is supported by National Science and Technology Key Special Projects (No. 2010ZX03003-002 & 2010ZX03003-004), National Nature Science Foundation of China (No. 60972023), and Research Fund of National Mobile Communications Research Laboratory, Southeast University (No. 2011A06), and also supported by UK-China Science Bridge: R&D of (B)4G Wireless Communications System.

Appendix A

Proof of Theorem 1 To simplify the rest of the derivations, let

$$\mathbf{Q} = \mathbf{W}\mathbf{P}\mathbf{P}^{\mathrm{H}}\mathbf{W}^{\mathrm{H}},$$

then $\mathbf{Q}$ is a positive definite matrix satisfying

$$\mathrm{Tr}(\mathbf{Q}) = \sum_{m=1}^{M} \|\omega_m\| \, p_m = \sum_{m=1}^{M} p_m = P. \tag{9}$$

Equation (5) can be written as

$$C = \max_{\mathrm{Tr}(\mathbf{Q})=P} \left\{ \log_2 \det\left(\mathbf{I}_N + \frac{1}{\sigma^2}\tilde{\mathbf{H}}\mathbf{Q}\tilde{\mathbf{H}}^{\mathrm{H}}\right)\right\}.$$

Since $(\tilde{\mathbf{H}}^{\mathrm{H}}\tilde{\mathbf{H}})_{ML\times ML}$ is a positive definite Hermitian matrix, using singular value decomposition (SVD), we have

$$\tilde{\mathbf{H}}^{\mathrm{H}}\tilde{\mathbf{H}} = \tilde{\mathbf{U}}^{\mathrm{H}}\tilde{\Lambda}\tilde{\mathbf{U}},$$

where $\tilde{\mathbf{U}}$ is a unitary matrix, and $\tilde{\Lambda}$ is a non-negative diagonal matrix whose diagonal elements are the eigenvalues of the matrix $\tilde{\mathbf{H}}^{\mathrm{H}}\tilde{\mathbf{H}}$, i.e., $\tilde{\Lambda} = \mathrm{diag}(\tilde{\lambda}_1, \tilde{\lambda}_2, \ldots, \tilde{\lambda}_{ML})$. And we obtain

$$\begin{aligned}\det\left(\mathbf{I}_N + \frac{1}{\sigma^2}\tilde{\mathbf{H}}\mathbf{Q}\tilde{\mathbf{H}}^{\mathrm{H}}\right) &= \det\left(\mathbf{I}_{ML} + \frac{1}{\sigma^2}\tilde{\mathbf{H}}^{\mathrm{H}}\tilde{\mathbf{H}}\mathbf{Q}\right) \\ &= \det\left(\mathbf{I}_{ML} + \frac{1}{\sigma^2}\tilde{\mathbf{U}}^{\mathrm{H}}\tilde{\Lambda}\tilde{\mathbf{U}}\mathbf{Q}\right) = \det\left(\mathbf{I}_{ML} + \frac{1}{\sigma^2}\tilde{\Lambda}\tilde{\mathbf{U}}\mathbf{Q}\tilde{\mathbf{U}}^{\mathrm{H}}\right).\end{aligned} \tag{10}$$

Let $\mathbf{A} = \tilde{\mathbf{U}}\mathbf{Q}\tilde{\mathbf{U}}^{\mathrm{H}}$, $\mathbf{B} = \mathbf{I}_{ML} + \frac{1}{\sigma^2}\tilde{\Lambda}\mathbf{A}$. Note that

$$\mathrm{Tr}(\mathbf{A}) = \mathrm{Tr}(\mathbf{Q}) = P.$$

Since $\mathbf{B}$ is a positive definite Hermitian matrix, from Hadamard theorem [12], we obtain

$$\det\left(\mathbf{I}_{ML} + \frac{1}{\sigma^2}\tilde{\Lambda}\tilde{\mathbf{U}}\mathbf{Q}\tilde{\mathbf{U}}^{\mathrm{H}}\right) \le \prod_{i=1}^{ML}\mathbf{B}_{ii} = \prod_{i=1}^{ML}\left(1 + \frac{1}{\sigma^2}\tilde{\lambda}_i\mathbf{A}_{ii}\right), \tag{11}$$

where $\mathbf{A}_{ii}$ and B_{ii} are diagonal elements of $\mathbf{A}$ and $\mathbf{B}$, respectively. In (11), there can be equality only if $\mathbf{B}$ is a diagonal matrix. If $\mathbf{A}$ is a diagonal matrix, $\mathbf{B}$ is a diagonal matrix. So Eq. (11) is equality only when $\mathbf{A}$ is a diagonal matrix. All diagonal elements of $\mathbf{A}$ are obtained by optimal waterfilling as

$$\mathbf{A}_{ii} = \begin{cases} 0 & \tilde{\lambda}_i = 0 \\ (\mu - \tilde{\lambda}_i^{-1})^+ & \tilde{\lambda}_i \neq 0 \end{cases}, \tag{12}$$

where μ is the waterfilling level and is chosen to satisfy

$$\sum_{i=1,\tilde{\lambda}_i \neq 0}^{ML} \left(\mu - \tilde{\lambda}_i^{-1}\right)^+ = P/\sigma^2 = \rho$$

and the resulting upper bound of the downlink capacity of the SA-MIMO system is

$$C \leq \sum_{i=1}^{ML} \left[\log_2\left(\mu\tilde{\lambda}_i\right)\right]^+$$

□

Appendix B

Proof of Theorem 2 In the case of equal transmit power allocation, i.e. $\mathbf{PP}^{\mathrm{H}} = \frac{P}{M}\mathbf{I}_M = \frac{\rho\sigma^2}{M}\mathbf{I}_M$, we can obtain the equivalent problem

$$\begin{aligned} C(\omega) = &\max_{\mathbf{w}} \left\{\log_2 \det\left(\mathbf{I}_N + \frac{\rho}{M}\mathbf{HH}^{\mathrm{H}}\right)\right\} \\ &\text{s.t.} \\ &\left\|\omega_j\right\|_{\mathrm{F}} = 1, \ \ j = 1, 2, \ldots, M. \end{aligned} \tag{13}$$

Let $\lambda_1, \lambda_2, \ldots, \lambda_R$ be non-zero eigenvalues of matrix $\mathbf{HH}^{\mathrm{H}}$. From the theorem of the arithmetic and geometric means, we obtain

$$\det\left(\mathbf{I}_N + \frac{\rho}{M}\mathbf{HH}^{\mathrm{H}}\right) = \prod_{i=1}^{R}\left(1 + \frac{\rho}{M}\lambda_i\right) \leq \frac{1}{R}\left(\sum_{i=1}^{R}\left(1 + \frac{\rho}{M}\lambda_i\right)\right)^R = \left(1 + \frac{\rho}{RM}\mathrm{Tr}\left(\mathbf{HH}^{\mathrm{H}}\right)\right)^R \tag{14}$$

with equality when

$$\lambda_1 = \lambda_2 = \cdots = \lambda_R. \tag{15}$$

From Rayleigh–Ritz theorem [12], we obtain

$$\mathrm{Tr}(\mathbf{HH}^{\mathrm{H}}) = \sum_{j=1}^{N} \omega_j^{\mathrm{H}}\left(\sum_{i=1}^{M}\left|h_{ij}\right|^2 \mathbf{a}_{ij}\mathbf{a}_{ij}^{\mathrm{H}}\right)\omega_j \leq \sum_{j=1}^{N}\lambda_{\max}(\mathbf{A}_j).$$

So the upper bound of the capacity of the SA-MIMO system with equal power allocation can be written as

$$C \leq R\log_2\left(1 + \frac{\rho}{RM}\sum_{j=1}^{N}\lambda_{\max}(\mathbf{A}_j)\right)$$

with equality when Eq. (15) is true. □

Appendix C

Proof of Theorem 3 Since the DOAs of all antennas at the receiver from one smart antenna array at the transmitter are same, the channel matrix $\mathbf{H}$ can be written as

$$\mathbf{H} = \bar{\mathbf{H}}\bar{\mathbf{W}}, \tag{16}$$

where

$$\bar{\mathbf{W}}_{M\times M} = \begin{bmatrix} \omega_1^H \mathbf{a}_1 & 0 & \\ & \ddots & \\ 0 & & \omega_M^H \mathbf{a}_M \end{bmatrix}.$$

In addition of equal transmit power allocation, the capacity is given by

$$\begin{aligned} C(\omega) &= \max_{\omega} \left\{ \log_2 \det \left(\frac{\rho}{M} \bar{\mathbf{H}}\bar{\mathbf{W}}\bar{\mathbf{W}}^{\mathrm{H}}\bar{\mathbf{H}}^{\mathrm{H}} + \mathbf{I}_N \right) \right\} \\ &\quad \text{s.t.} \\ &\quad \|\omega_m\| = 1,\ m = 1, 2, \ldots, M \qquad (17) \\ &= \max_{\omega} \left\{ \log_2 \det \left(\frac{\rho}{M} \bar{\mathbf{W}}^{\mathrm{H}}\bar{\mathbf{H}}^{\mathrm{H}}\bar{\mathbf{H}}\bar{W} + \mathbf{I}_M \right) \right\} \\ &\quad \text{s.t.} \\ &\quad \|\omega_m\| = 1,\ m = 1, 2, \ldots, M. \qquad (18) \end{aligned}$$

Using singular value decomposition (SVD), we have

$$\bar{\mathbf{H}}^{\mathrm{H}}\bar{\mathbf{H}} = \bar{\mathbf{U}}^{\mathrm{H}}\bar{\Lambda}\bar{\mathbf{U}}, \tag{19}$$

where $\bar{\mathbf{U}}$ is a unitary matrix, and $\bar{\Lambda}$ is a non-negative diagonal matrix whose diagonal elements are the singular values of the matrix $\bar{\mathbf{H}}^{\mathrm{H}}\bar{\mathbf{H}}$, i.e., $\bar{\Lambda} = \mathrm{diag}(\bar{\lambda}_1, \bar{\lambda}_2, \ldots, \bar{\lambda}_M)$. So we obtain

$$\begin{aligned} &\log_2 \det \left(\frac{\rho}{M} \bar{\mathbf{W}}^{\mathrm{H}}\bar{\mathbf{H}}^{\mathrm{H}}\bar{\mathbf{H}}\bar{\mathbf{W}} + \mathbf{I}_M \right) \\ &= \log_2 \det \left(\frac{\rho}{M} \bar{\mathbf{W}}^{\mathrm{H}}\bar{\mathbf{U}}^{\mathrm{H}}\bar{\Lambda}\bar{\mathbf{U}}\bar{\mathbf{W}} + \mathbf{I}_M \right) \\ &= \log_2 \det \left(\frac{\rho}{M} \bar{\Lambda}\bar{\mathbf{U}}\bar{\mathbf{W}}\bar{\mathbf{W}}^{\mathrm{H}}\bar{\mathbf{U}}^{\mathrm{H}} + \mathbf{I}_M \right). \end{aligned} \tag{20}$$

To simplify the rest of the derivations, let

$$\mathbf{A} = \bar{\mathbf{U}}\bar{\mathbf{W}}\bar{\mathbf{W}}^{\mathrm{H}}\bar{\mathbf{U}}^{\mathrm{H}}, \mathbf{B} = \frac{\rho}{M}\bar{\Lambda}\mathbf{A} + \mathbf{I}_M.$$

Since $\mathbf{B}$ is a positive definite Hermitian matrix, from Hadamard theorem 12, we obtain

$$\log_2 \det \left(\frac{\rho}{N} \bar{\Lambda}\bar{\mathbf{U}}\bar{\mathbf{W}}\bar{\mathbf{W}}^{\mathrm{H}}\bar{\mathbf{U}}^{\mathrm{H}} + \mathbf{I}_M \right) \le \log_2 \prod_{i=1}^{M} \mathbf{B}_{ii} = \log_2 \prod_{i=1}^{M} \left(1 + \bar{\lambda}_i \frac{\rho}{N} \mathbf{A}_{ii} \right), \tag{21}$$

where $\mathbf{A}_{ii}$ and B_{ii} are diagonal elements of $\mathbf{A}$ and $\mathbf{B}$, respectively. In (21), there can be equality only if $\mathbf{B}$ is a diagonal matrix. If $\mathbf{A}$ is a diagonal matrix, $\mathbf{B}$ is a diagonal matrix. So Eq. (21) is established only when $\mathbf{A}$ is a diagonal matrix. According to the expression of $\mathbf{A}$, it can be known that all diagonal elements are independent. Therefore

$$\mathbf{A}_{ii} = \sum_{j=1}^{M} \left| \omega_j^{\mathrm{H}} \mathbf{a}_j \right|^2 |u_{ij}|^2 \le \sum_{j=1}^{M} \left\| \omega_j^{\mathrm{H}} \right\|^2 \|\mathbf{a}_j\|^2 |u_{ij}|^2 = L \sum_{j=1}^{M} |u_{ij}|^2 = L. \tag{22}$$

In (22), there can be equality if and only if ω_i is given by

$$\omega_i = \frac{1}{\sqrt{L}}\mathbf{a}_i, i = 1, 2, \ldots, M. \tag{23}$$

And after substitution, the capacity can be written as

$$C = \sum_{i=1}^{M} \log_2\left(1 + \frac{\rho}{M}\bar{\lambda}_i L\right),$$

if and only if the beam-forming vector is given by (23). □

References

1. Chen, H.-M., Wang, J.-B., & Chen, M. (2011). Outage capacity study of the distributed MIMO system with antenna cooperation. *Wireless Personal Communications, 59*(4), 599–605.
2. Wang, J.-B., Wang, J.-Y., & Chen, M. (2011). Downlink system capacity analysis in distributed antenna systems. *Wireless Personal Communications* (online first). September 13 2011.
3. Michailidis, E. T., & Kanatas, A. G. (2011). Statistical simulation modeling of 3-D HAP-MIMO channels. *Wireless Personal Communications* (online first). April 10 2011.
4. Wang, P., Zu, L., Liu, F., Han, J., Ai, Q., et al. (2011). Downlink performance analysis of MIMO relaying networks. *Wireless Personal Communications* (online first). September 19 2011.
5. Wang, C.-X., Hong, X., Ge, X., Cheng, X., Zhang, G., & Thompson, J. (2010). Cooperative MIMO channel models: A survey. *IEEE Communications Magazine, 48*(2), 80–87.
6. Telatar, E. (1999). Capacity of multi-antenna Gaussian channels. *European Transactions on Telecommunications, 10*(6), 585–595.
7. Foschini, G. J., & Gans, M. J. (1998). On limits of wireless communications in a fading environment when using multiple antennas. *Wireless Personal Communications, 6*(2), 311–335.
8. Paulraj, A. J., Gore, D. A., Nabar, R. U., & Bolcskei, H. (2004). An overview of MIMO communications—a key to gigabit wireless. *Proceedings of the IEEE, 92*(2), 198–218.
9. Andersen, J. B. (2000). Array gain and capacity for known random channels with multiple element arrays at both ends. *IEEE Journal on Selected Areas in Communication, 18*, 2172–2178.
10. Dakdouki, A. S., & Tabulo, M. (2004). On the eigenvalue distribution of smart-antenna arrays in wireless communication systems. *IEEE Antennas and Propagation Magazine, 46*(4), 158–167.
11. Cover, T. M., & Thomas, J. A. (1991). *Elements of information theory*. New York, NY: Wiley.
12. Bellmam, R. (1970). *Introduction to matrix analysis*. New York, NY: Mcgraw-Hillbook company.

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