

Impact of Arbitrary Co-channel MIMO Modes on Alamouti Coding Under Path-Loss and Rayleigh Fading

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Abstract—Co-channel interference has a tremendous impact on cellular-like systems, especially when MIMO techniques are incorporated. Taking into account the effects of both path-loss and Rayleigh fading, we evaluate the impact of a mix of co-channel MIMO interferers on the performance of Alamouti Coding. In particular, we derive closed-form expressions for the probability density functions of the resulting signal-to-interference ratios. Simulation results verify the validity of the analyses.

I. INTRODUCTION

MIMO, utilizing multiple antennas at both the transmitter and receiver, has emerged as a cornerstone of modern wireless communications. As an assignable resource, the MIMO mode [1] (such as, Space-Time Block Coding (STBC), open-loop spatial multiplexing (OLSM), closed-loop spatial multiplexing (CLSM), MIMO beamforming (MBF), maximal ratio transmission (MRT) and single-input multiple-output (SIMO), etc.) can be switched to adapt to the variations in the instantaneous link quality. Among these MIMO modes, due to its superior ability in combating fading and its robustness to co-channel interference (CCI), STBC [2][3] is a good option to communicate with users on the cell edges in interference-limited systems. The focus of this paper, *Alamouti Coding* [2] which is the simplest form of STBC, has been adopted in several emerging wireless standards. Since STBC uses multiple symbol-periods to recover the transmitting symbols, the effect of CCI on STBC is quite different from that of other MIMO schemes in the desired link.

The signal-to-interference-plus-noise ratio (SINR) or signal-to-interference ratio (SIR) distribution is often used as the metric to investigate the performance of an interference-limited system. The SNR/SINR distribution of maximal ratio combining (MRC) and optimal combining with CCI are discussed in [4][5] and [6], respectively. When the desired base station (BS) employs MBF or MRT, some work has been done in determining the statistical distribution of the post-processing SIR/SINR [7][8].

The distributions of the post-processing SINR for an STBC receiver, which is the focus of this paper, were first derived in [9]. This work assumes that all the BSs adopt the same MIMO mode and transmitting power. To relax this constraint, in [10], the impact of other interfering MIMO schemes on STBC was studied; however, only a simulation-based approach is used. In [11], the closed-form probability density function (*pdf*)

expressions for the SIR are derived for STBC with different MIMO modes under the assumption that the interfering BSs transmit with equal power.

Here, taking into account the effects of path-loss, Rayleigh fading, and possible unequal transmit powers, the distributions of the post-processing SIRs (noise is neglected here since we are considering an interference-limited environment) for STBC with arbitrary co-channel MIMO modes are investigated. In particular, closed-form *pdf* expressions for the interference terms in the post-processing SIRs are provided. The validity of the theoretical analyses is verified by simulation results.

The rest of the paper is organized as follows. The system model and assumptions are described in Section II. We analyze the SIRs in Section III and then present the corresponding distributions in Section IV. Simulation results and conclusions are provided in Sections V and VI, respectively.

II. SYSTEM MODEL AND ASSUMPTIONS

Here, we focus on the downlink of a cellular-like system, and consider an interference-limited system using STBC in the desired link with K active co-channel interfering BSs. Each transmitter and the desired receiver are assumed to be equipped with N and M antennas, respectively. Denote $h_{k,ji}$ ($k = 0, 1, \dots, K$; $j = 1, \dots, M$; $i = 1, \dots, N$) as the instantaneous channel gain between the j^{th} receive antenna at the desired user and the i^{th} transmit antenna at the k^{th} BS. The index $k = 0$ indicates the desired BS.

Including both the effects of path-loss and Rayleigh fading in the variances of the channel coefficients, we model $\{h_{k,ji}\}$ as independent zero-mean complex Gaussian random variables (RV) with variance μ_k . Since the distances from the antennas of a co-located array to the antennas of another co-located array can be considered equal, for any given k , the coefficients $\{h_{k,ji}\}$ have the same mean channel gain μ_k . Moreover, the transmit power of *each antenna* * at the k^{th} BS is denoted as P_k . For simplicity, we include the transmit powers in the channel power gains; so, we consider an equivalent system in which (i) the channels $\{h_{k,ji}\}$ are independent zero-mean complex Gaussian RVs with variance $P_k\mu_k$ and (ii) each antenna at each BS has unit transmit power. Here, $P_k\mu_k$ is the

* All BSs have the same total transmit power constraint, i.e., $NP_k \leq P$, where P is a constant. Since transmit power is an assignable resource, different BSs may have different transmit powers.

average received power at one receive antenna (of the desired user) from one transmit antenna (of the k^{th} BS). Since path-loss and transmit power may be distinct for different BSs, $P_i\mu_i$ might not be equal to $P_j\mu_j$ ($i \neq j$).

Each BS is assumed to only have knowledge about its own channel state information. When a BS (including the desired BS) uses STBC, Alamouti Coding[†] [2] is adopted. To be fair, all the BSs are assumed to have two antennas, i.e., $N = 2$. For every two consecutive symbol periods, during the first symbol period, $s_{k,1}$ and $s_{k,2}$ are transmitted from the two antennas of the k^{th} BS. During the subsequent symbol period, $s_{k,3}$ and $s_{k,4}$ are sent simultaneously. Based on the above assumptions, the baseband signal vector received at the j^{th} antenna of the desired user is

$$\mathbf{r}_j = \underbrace{\mathbf{s}_0 \mathbf{W}_0 \mathbf{h}_{0,j}}_{\text{desired signal}} + \underbrace{\sum_{k=1}^K \mathbf{s}_k \mathbf{W}_k \mathbf{h}_{k,j}}_{\text{CCI}} \quad (1)$$

The notation in (1) is as follows: $\mathbf{r}_j = [r_{j,1} \ r_{j,2}]^T$ is the received signal vector. $\mathbf{h}_{k,j} = [h_{k,j1} \ h_{k,j2}]^T$ denotes the channels between the j^{th} receiving antenna of the desired user and the two transmitting antennas of the k^{th} BS. Moreover, the signal matrix and the corresponding precoding matrix, which depend on the specific MIMO mode, are

$$\mathbf{s}_k = \begin{bmatrix} s_{k,1} & s_{k,2} \\ s_{k,3} & s_{k,4} \end{bmatrix} \quad \text{and} \quad \mathbf{W}_k = \begin{bmatrix} w_{k,11} & w_{k,12} \\ w_{k,21} & w_{k,22} \end{bmatrix} \quad (2)$$

respectively. For the desired link using Alamouti Coding,

$$\mathbf{s}_0 = \begin{bmatrix} s_{0,1} & s_{0,2} \\ -s_{0,2}^* & s_{0,1}^* \end{bmatrix} \quad (3)$$

and the precoding matrix \mathbf{W}_0 is an identity matrix.

III. POST-PROCESSING SIR EXPRESSIONS

In the interfering BSs, six commonly used MIMO modes can be used which, based on the results in [11], are divided into two categories:

- Category I: STBC, OLSM and CLSM
- Category II: MBF, MRT and SIMO

For Category I MIMO schemes, multiple independent data streams are sent simultaneously during one symbol period; on the other hand, for Category II, only a single data stream is sent during one period. In prior work [11], we show that the co-channel MIMO interfering schemes of the same category have the same effect on the desired received signal from the perspective of the STBC receiver. It should be noted that spatial multiplexing and STBC used in the interfering BSs have similar impact on the desired received signals [11].

[†]Alamouti Coding is full-rate [2] and has been adopted in several current wireless standards. In contrast, high-order STBCs [3] only have a maximum possible rate of 3/4 and have very long block-lengths which makes them unsuitable for practical use. It should be noted that the distribution of the SIR for higher-order STBCs [3] can be derived in a similar manner.

A. Interfering BSs Use Category I MIMO Modes

Here, we use the notations of STBC to derive the SIR expression for Category I MIMO modes. When all the interfering links use STBC, \mathbf{W}_k reduces to an identity matrix; and, the signal matrix at the k^{th} BS is

$$\mathbf{s}_k = \begin{bmatrix} s_{k,1} & s_{k,2} \\ -s_{k,2}^* & s_{k,1}^* \end{bmatrix} \quad (4)$$

Denote

$$\begin{aligned} \mathbf{a}_0 &= [h_{0,11}^* \ h_{0,12} \ \cdots \ h_{0,M1}^* \ h_{0,M2}]^T \\ \mathbf{a}_{k,1} &= [h_{k,11} \ h_{k,12}^* \ \cdots \ h_{k,M1}^* \ h_{k,M2}^*]^T \\ \mathbf{a}_{k,2} &= [h_{k,12} \ -h_{k,11}^* \ \cdots \ h_{k,M2}^* \ -h_{k,M1}^*]^T \end{aligned}$$

Then, the post-processing SIR at the desired receiver with Category I MIMO modes is [11]

$$\gamma_{\text{STBC, I}} = \frac{x_0}{\sum_{k=1}^K \{y_{k,1} + y_{k,2}\}} \quad (5)$$

where $x_0 \triangleq |\mathbf{a}_0^H \mathbf{a}_0|$ and $y_{k,i} \triangleq |\mathbf{a}_0^H \mathbf{a}_{k,i}|^2 / |\mathbf{a}_0^H \mathbf{a}_0|$ ($i = 1, 2$), respectively. Clearly, the numerator x_0 has a central chi-square distribution with pdf $f_{x_0}(x_0) = \frac{x_0^{2M-1} \exp\{-x_0/(P_0\mu_0)\}}{(P_0\mu_0)^{2M} \Gamma(2M)}$. From [4], we know that $y_{k,i}$ has an exponential distribution with pdf $f_{y_{k,i}}(y_{k,i}) = \frac{1}{P_k\mu_k} \exp\{-y_{k,i}/(P_k\mu_k)\}$. For any given $k \in \{1, \dots, K\}$, it is reasonable to assume that $y_{k,1}$ and $y_{k,2}$ are mutually independent [9][11]. In general, for $k' \neq k''$, because $\mu_{k'} \neq \mu_{k''}$ and $P_{k'} \neq P_{k''}$, $y_{k',i}$ and $y_{k'',i}$ ($i = 1, 2$) are independent but have different distributions, except for the special case when $P_{k'}\mu_{k'} = P_{k''}\mu_{k''}$. In contrast, for any given k , $y_{k,1}$ and $y_{k,2}$ have the same distribution. Thus, generally, $\sum_{k=1}^K (y_{k,1} + y_{k,2})$ is the sum of $2K$ independent, but non-identical exponential RVs.

As in [9], although the numerator and the denominator in (5) may not be independent since both include the terms related to \mathbf{a}_0 , here, we will assume that they are independent. This assumption can be justified from the fact that the common term $|\mathbf{a}_0^H|^2 / |\mathbf{a}_0^H \mathbf{a}_0|$ is multiplied by independent RVs, so they become nearly independent. The assumption will be applied in the subsequent derivations, and justified later through Monte-Carlo simulations.

B. Interfering BSs Use Category II MIMO Modes

SIMO is used to derive the SIR expression when Category II MIMO modes are adopted in the interfering links. In the case of SIMO (i.e., receive diversity), only one antenna is used at the interfering BSs and no precoding is needed. To be consistent with other two-antenna transmission modes, we use the following equivalent two-antenna signal matrix and channel vector,

$$\mathbf{s}_k = \begin{bmatrix} \frac{\sqrt{2}}{2} s_{k,1} & \frac{\sqrt{2}}{2} s_{k,1} \\ \frac{\sqrt{2}}{2} s_{k,2} & \frac{\sqrt{2}}{2} s_{k,2} \end{bmatrix} \quad \text{and} \quad \mathbf{h}_{k,j} = [h_{k,j1} \ h_{k,j1}]^T \quad (6)$$

for the k^{th} interfering BS which uses SIMO transmission. With this equivalent model, SIMO can be also treated as a two-antenna transmission mode. Denote

$$\mathbf{d}_{0,i} = [h_{0,1i} \cdots h_{0,Mi}]^T, \quad \mathbf{d}_k = [h_{k,11} \cdots h_{k,M1}]^T$$

Then, the post-processing SIR at the desired receiver with Category II MIMO modes is [11]

$$\gamma_{\text{STBC, II}} \approx \frac{x_0}{2 \cdot \sum_{k=1}^K z_k} \quad (7)$$

where $x_0 \triangleq \sum_{i=1}^2 |\mathbf{d}_{0,i}^H \mathbf{d}_{0,i}|$ which has the same distribution as the numerator in (5); and $z_k \triangleq |\mathbf{d}_{0,i}^H \mathbf{d}_k|^2 / |\mathbf{d}_{0,i}^H \mathbf{d}_{0,i}|$. Similar to Section III-A, z_k has an exponential distribution [4] with pdf $f_{z_k}(z_k) = \frac{1}{P_k \mu_k} \exp\{-z_k/(P_k \mu_k)\}$; and, the numerator and the denominator in (7) are assumed to be independent.

C. Interfering BSs Use Arbitrary MIMO Modes

Here, we consider the general scenario in which the interfering BSs adopt arbitrary MIMO modes. In other words, the desired received signal is corrupted by a mix of co-channel MIMO interferences. Without loss of generality, among K co-channel MIMO interfering BSs, assume that the first K_1 BSs use Category I MIMO modes and the remaining $K_2 = K - K_1$ BSs adopt Category II. Based on the above assumptions and combining (5) with (7), the post-processing SIR of the desired STBC receiver with K co-channel interferers is

$$\gamma_{\text{STBC}} \approx \frac{x_0}{\sum_{k=1}^{K_1} \{y_{k,1} + y_{k,2}\} + 2 \cdot \sum_{k=K_1+1}^K z_k} \triangleq \frac{X}{Y} \quad (8)$$

where $X \triangleq x_0$ and $Y \triangleq \sum_{k=1}^{K_1} \{y_{k,1} + y_{k,2}\} + 2 \cdot \sum_{k=K_1+1}^K z_k$, respectively. Specifically, X has a central chi-square distribution; $y_{k,i}$ and z_k are exponential RVs.

Generally, the denominator Y in (8) is the sum of $2K_1 + K_2$ independent but non-identically exponentially distributed RVs. Only when the interfering links use the same category MIMO mode with equal power/path-loss (i.e., (i) $K_2 = 0$ with $P_1 \mu_1 = \cdots = P_K \mu_K$, or (ii) $K_1 = 0$ with $P_{K_1+1} \mu_{K_1+1} = \cdots = P_K \mu_K$), can Y be simplified as the sum of i.i.d. exponentially distributed RVs. Furthermore, as in Section III-A, we assume X and Y to be independent.

IV. DISTRIBUTION OF POST-PROCESSING SIR

We will first derive the closed-form pdf expression for Y , then analyze the distribution of the post-processing SIR. For simplicity, we let

$$w_k = 2z_k \quad (9)$$

Obviously, w_k is an exponential RV with pdf $f_{w_k}(w_k) = \frac{1}{2P_k \mu_k} \exp\{-w_k/(2P_k \mu_k)\}$. Denote α_l ($l = 1, \dots, 2K_1 + K_2$) as independent zero-mean complex Gaussian RVs with per-dimension variances $P_k \mu_k$ ($k = \lceil l/2 \rceil$) for $l = 1, \dots, 2K_1$ and $2P_k \mu_k$ ($k = l - K_1$) for $l = 2K_1 + 1, \dots, 2K_1 + K_2$, where

$\lceil l/2 \rceil$ means ‘‘rounding $l/2$ to the the next largest integer’’. Then

$$\begin{aligned} y_{k,i} &= |\alpha_{2(k-1)+i}|^2 & k = 1, \dots, K_1; \quad i = 1, 2 \\ w_k &= |\alpha_{k+K_1}|^2 & k = K_1 + 1, \dots, K \end{aligned} \quad (10)$$

Let $\boldsymbol{\alpha} = [\alpha_1, \dots, \alpha_{2K_1+K_2}]^T$ and denote the $(2K_1 + K_2) \times (2K_1 + K_2)$ identity matrix as \mathbf{I} . Since \mathbf{I} is a semi-positive definite Hermitian matrix, and $\boldsymbol{\alpha}$ is a zero-mean complex Gaussian random vector, $Y \triangleq \sum_{k=1}^{K_1} (y_{k,1} + y_{k,2}) + \sum_{k=K_1+1}^K w_k = \sum_{i=1}^{2K_1+K_2} |\alpha_i|^2 = \boldsymbol{\alpha}^H \boldsymbol{\alpha} = \boldsymbol{\alpha}^H \mathbf{I} \boldsymbol{\alpha}$ is a central quadratic form in complex Gaussian RVs [12]. Moreover, denote the covariance matrix of $\boldsymbol{\alpha}$ as $\mathbf{R} \in \mathbb{C}^{(2K_1+K_2) \times (2K_1+K_2)}$,

$$\begin{aligned} \mathbf{R} &= \mathbb{E}[\boldsymbol{\alpha} \boldsymbol{\alpha}^H] \\ &= \text{diag}\{P_1 \mu_1, P_1 \mu_1, \dots, P_{K_1} \mu_{K_1}, P_{K_1} \mu_{K_1}, \\ &\quad 2P_{K_1+1} \mu_{K_1+1}, \dots, 2P_K \mu_K\} \end{aligned} \quad (11)$$

Obviously, \mathbf{R} is a semi-positive definite real symmetric matrix. Hence, there exists a real symmetric matrix $\mathbf{Q} \in \mathbb{C}^{(2K_1+K_2) \times (2K_1+K_2)}$ for which $\mathbf{Q}\mathbf{Q} = \mathbf{R}$. Then, we get

$$\begin{aligned} \mathbf{Q} &= \text{diag}\{\sqrt{P_1 \mu_1}, \sqrt{P_1 \mu_1}, \dots, \sqrt{P_{K_1} \mu_{K_1}}, \sqrt{P_{K_1} \mu_{K_1}}, \\ &\quad \sqrt{2P_{K_1+1} \mu_{K_1+1}}, \dots, \sqrt{2P_K \mu_K}\} \end{aligned} \quad (12)$$

Let $\mathbf{B} = \mathbf{Q}\mathbf{I}\mathbf{Q}$, which is a semi-positive definite Hermitian matrix. Since $\mathbf{B} = \mathbf{Q}\mathbf{I}\mathbf{Q} = \mathbf{R}$, the eigenvalue decompositions of \mathbf{B} and \mathbf{R} are equivalent. In other words, the $2K_1 + K_2$ eigenvalues of \mathbf{B} are

$$\begin{aligned} \lambda_{2(k-1)+i} &= P_k \mu_k & k = 1, \dots, K_1; \quad i = 1, 2 \\ \lambda_{k+K_1} &= 2P_k \mu_k & k = K_1 + 1, \dots, K \end{aligned} \quad (13)$$

Without loss of generality, denote M_p ($M_p \leq 2K_1 + K_2$) as the total number of distinct eigenvalues of \mathbf{B} and v_m as the multiplicity of λ_m ($m = 1, \dots, M_p$). For central quadratic forms in complex Gaussian RVs, the moment generating function (MGF) of Y is

$$\Phi_Y(s) = \mathbb{E}[e^{sY}] = 1 / \prod_{m=1}^{M_p} (1 - s\lambda_m)^{v_m} \quad (14)$$

Let $f_Y(y)$ be the pdf of Y , which is the inverse Laplace transform of $\Phi_Y(s)$. Using the Laplace Inversion Theorem, Jordan’s Lemma, and the Residue Theorem [13], we get

$$f_Y(y) = - \sum_{m=1}^{M_p} f_m(y) = - \sum_{m=1}^{M_p} \text{Res}[\Phi_Y(s) e^{-sy}, 1/\lambda_m] \quad (15)$$

where $f_m(y)$ is the residue of $\Phi_Y(s) e^{-sy}$ at the pole $1/\lambda_m$.

$f_m(y)$ ($m = 1, \dots, M_p$) is computed as follows,

$$\begin{aligned} f_m(y) &= \text{Res}[\Phi_Y(s)e^{-sy}, 1/\lambda_m] \\ &= \frac{1}{\Gamma(v_m)} \lim_{s \rightarrow \frac{1}{\lambda_m}} \frac{d^{v_m-1}}{ds^{v_m-1}} \left[\Phi_Y(s)e^{-sy} \left(s - \frac{1}{\lambda_m} \right)^{v_m} \right] \\ &= \frac{(-\lambda_m)^{-v_m}}{\Gamma(v_m)} e^{-\frac{y}{\lambda_m}} \sum_{r_m=0}^{v_m-1} C_{v_m-1}^{r_m} (-y)^{v_m-1-r_m} \phi^{(r_m)}(\lambda_m) \end{aligned} \quad (16)$$

where $C_t^i = t!/[i!(t-i)!]$ denotes the number of ways of selecting i elements from t elements; $C_t^i = 1$ for $i = 0$ and $t \geq 0$. Moreover, $\phi^{(r_m)}(\lambda_m)$ is the r_m^{th} order derivative of $\phi(\lambda_m)$, with $\phi(\lambda_m)$ defined as

$$\phi(\lambda_m) = \begin{cases} 1, & M_p = 1 \\ \prod_{\substack{n=1 \\ n \neq m}}^{M_p} \left(\frac{\lambda_m}{\lambda_m - \lambda_n} \right)^{v_n}, & M_p > 1 \end{cases} \quad (17)$$

The derivatives of $\phi(\lambda_m)$ are calculated as [12]

$$\phi^{(r_m)}(\lambda_m) = G^{(r_m)}(\lambda_m)\phi(\lambda_m) \quad (18)$$

where $G^{(r_m)}(\lambda_m)$ ($m = 1, \dots, M_p$) is equal to 1 when $r_m = 0$, and is equal to 0 when $r_m \geq 1$ and $M_p = 1$; when $r_m \geq 1$ and $M_p > 1$, it is given as

$$\sum_{r'_m=0}^{r_m-1} C_{r_m-1}^{r'_m} g^{(r_m-1-r'_m)}(\lambda_m) \sum_{r''_m=0}^{r'_m-1} C_{r'_m-1}^{r''_m} g^{(r'_m-1-r''_m)}(\lambda_m) \times \dots \quad (19)$$

In (19), $g^{(t)}(\lambda_m)$ ($m = 1, \dots, M_p$) is further specified as

$$g^{(t)}(\lambda_m) = t! \times \sum_{\substack{n=1 \\ n \neq m}}^{M_p} \frac{v_n \times \lambda_n^{t+1} \times \lambda_m^{t+1}}{(\lambda_m - \lambda_n)^{t+1}} \quad (20)$$

By combining (16)-(20), the closed-form expression for the pdf of Y is given as

$$\begin{aligned} f_Y(y) &= \sum_{m=1}^{M_p} \left\{ \frac{e^{-\frac{y}{\lambda_m}} \phi(\lambda_m)}{(-\lambda_m)^{v_m} \Gamma(v_m)} \sum_{r_m=0}^{v_m-1} C_{v_m-1}^{r_m} (-y)^{v_m-1-r_m} G^{(r_m)}(\lambda_m) \right\} \end{aligned} \quad (21)$$

Then, the pdf of the post-processing SIR, $\gamma = X/Y$ ($\gamma > 0$), is given as [14]

$$f_\gamma(\gamma) = \int_{-\infty}^{+\infty} f_X(y\gamma) f_Y(y) |y| dy = \int_0^{+\infty} f_X(y\gamma) f_Y(y) y dy \quad (22)$$

Due to the complicated mathematical form of $f_Y(y)$ shown in (21), the closed-form expression of (22) is not easy to obtain. In general, numerical computations can be applied to evaluate it. However, in some special cases which will be shown subsequently, $f_Y(y)$ can be simplified, and a closed-form expression for $f_\gamma(\gamma)$ can be obtained.

1) *All the interfering BSs Use Category II with Distinct Power/Path-loss*: In this scenario, we get $K_1 = 0$ and $P_i \mu_i \neq P_j \mu_j$ for $i \neq j$ with $i, j \in \{1, \dots, K\}$. Moreover, we know that $M_p = K$ and $\lambda_k = 2P_k \mu_k$ with $v_k = 1$ ($k = 1, \dots, K = M_p$). Then, using (17) and (19)-(21), we have

$$f_Y(y) = \sum_{k=1}^K \left\{ \frac{e^{-\frac{y}{2P_k \mu_k}}}{2P_k \mu_k} \prod_{\substack{j=1 \\ j \neq k}}^K \frac{P_k \mu_k}{P_k \mu_k - P_j \mu_j} \right\} \quad (23)$$

Denote the cumulative distribution function (CDF) of the SIR, $\gamma = X/Y$ ($\gamma \geq 0$), as $F_\gamma(r) = \Pr\{\gamma < r\}$. Then

$$\begin{aligned} F_\gamma(r) &= \Pr\{X/Y < r\} = \Pr\{Y > X/r\} \\ &= \int_0^{+\infty} f_X(x) \int_{x/r}^{+\infty} f_Y(y) dy dx \end{aligned} \quad (24)$$

Because $X \triangleq x_0 = \sum_{i=1}^2 \sum_{j=1}^M |h_{0,ji}|^2$ is also a central quadratic form in i.i.d. complex Gaussian RVs with zero mean and variance $P_0 \mu_0$, for X , $M_p = 1$ and $\lambda_1 = P_0 \mu_0$ with $v_m = 2M$. From (14), we get the MGF of X as follows,

$$\Phi_X(s) = E[e^{sx}] = \frac{1}{(1 - s \times P_0 \mu_0)^{2M}} \quad (25)$$

Furthermore, we denote $\prod_{j=1, j \neq k}^K \frac{P_k \mu_k}{P_k \mu_k - P_j \mu_j}$ as ξ_k ; by utilizing the fact that $\int_0^{+\infty} f_Y(y) dy = 1$, we can show that $\sum_{k=1}^K \xi_k = 1$. Then, using (23), (24) becomes

$$\begin{aligned} F_\gamma(\gamma) &= \int_0^{+\infty} f_X(x) \int_{x/\gamma}^{+\infty} \sum_{k=1}^K \frac{\xi_k}{2P_k \mu_k} \exp\left(-\frac{y}{2P_k \mu_k}\right) dy dx \\ &= \int_0^{+\infty} f_X(x) \sum_{k=1}^K \xi_k \exp\left(-\frac{x}{2\gamma P_k \mu_k}\right) dx \\ &= \sum_{k=1}^K \xi_k \int_0^{+\infty} \exp\left(-\frac{1}{2\gamma P_k \mu_k} x\right) f_X(x) dx \\ &= \sum_{k=1}^K \xi_k \cdot E \left[\exp\left(-\frac{1}{2\gamma P_k \mu_k} x\right) \right] \end{aligned} \quad (26)$$

By replacing (25) with $s = -1/(2\gamma P_k \mu_k)$, the CDF of the post-processing SIR is

$$F_\gamma(\gamma) = \sum_{k=1}^K \frac{\xi_k}{\left(1 + \frac{1}{2\gamma} \frac{P_0 \mu_0}{P_k \mu_k}\right)^{2M}} \quad (27)$$

By differentiating (27), we get the pdf of the SIR as

$$f_\gamma(\gamma) = \sum_{k=1}^K \xi_k \cdot \left(\frac{2P_k \mu_k}{P_0 \mu_0} \right) \cdot \frac{(2M) \left(\frac{2P_k \mu_k}{P_0 \mu_0} \gamma \right)^{2M-1}}{\left(1 + \frac{2P_k \mu_k}{P_0 \mu_0} \gamma\right)^{2M+1}}, \quad \gamma > 0 \quad (28)$$

2) *All the Interfering BSs Use Category I with Equal Power/Path-loss:* In this case, we get $K_2 = 0$, $P_1\mu_1 = \dots = P_K\mu_K$, $M_p = 1$ and $\lambda_1 = P_1\mu_1$ with multiplicity $v_1 = 2K$. Then, using (17) and (19)-(21), we get

$$f_Y(y) = \frac{y^{2K-1} \exp\left(\frac{-y}{P_1\mu_1}\right)}{(P_1\mu_1)^{2K} \Gamma(2K)} \quad (29)$$

This is a central chi-square distribution with $2 \times 2K$ degrees of freedom, consistent with what was obtained in [11] for equal-power STBC interferers with i.i.d. channels. The SIR, $\gamma = X/Y$, follows a canonical F_C distribution with pdf [11]

$$f_\gamma(\gamma) = \left(\frac{P_1\mu_1}{P_0\mu_0}\right) \cdot \frac{\Gamma(2M+2K) \left(\frac{P_1\mu_1}{P_0\mu_0}\gamma\right)^{2M-1}}{\Gamma(2M)\Gamma(2K) \left(1 + \frac{P_1\mu_1}{P_0\mu_0}\gamma\right)^{2M+2K}}, \quad \gamma > 0 \quad (30)$$

3) *All the Interfering BSs Use Category II with Equal Power/Path-loss:* In this scenario, we have $K_1 = 0$, $P_1\mu_1 = \dots = P_K\mu_K$, $M_p = 1$ and $\lambda_1 = 2P_1\mu_1$ with multiplicity $v_1 = K$. By exploiting (17) and (19)-(21), the pdf of Y is

$$f_Y(y) = \frac{y^{K-1} \exp\left(\frac{-y}{2P_1\mu_1}\right)}{(2P_1\mu_1)^K \Gamma(K)} \quad (31)$$

In particular, this is a central chi-square distribution with $2 \times K$ degrees of freedom, again consistent with what we derived in [11] for equal-power STBC interferers with i.i.d. channels. The pdf for the SIR has the form as

$$f_\gamma(\gamma) = \left(\frac{2P_1\mu_1}{P_0\mu_0}\right) \cdot \frac{\Gamma(2M+K) \left(\frac{2P_1\mu_1}{P_0\mu_0}\gamma\right)^{2M-1}}{\Gamma(2M)\Gamma(K) \left(1 + \frac{2P_1\mu_1}{P_0\mu_0}\gamma\right)^{2M+K}}, \quad \gamma > 0 \quad (32)$$

V. SIMULATION RESULTS

In this section, the validity of the theoretical analyses is verified by comparing the analytically obtained CDFs with Monte-Carlo simulations. Alamouti Coding is adopted at the desired BSs; hence, all the BSs are assumed to be equipped with two antennas. Since the co-channel MIMO interfering schemes of the same category have the same effect on the desired received signal [11], STBC and SIMO[‡] are used to represent Category I and Category II interferers, respectively. For the desired link, $P_0\mu_0$ is set to 1. The notation $K = 6, K_1 = 3\{1, 0.9, 0.8, 0.7, 0.6, 0.5\}$ means that there are $K = 6$ interfering BSs, among which the first $K_1 = 3$ interfering BSs adopt Category I MIMO modes and the other $K - K_1 = 3$ use Category II; the k^{th} number within the braces stands for the value of $P_k\mu_k$ for the k^{th} interfering BS. Moreover, two antenna configurations, (2Tx, 1Rx) and (2Tx, 2Rx) are considered.

In Figs. 1-4, we verify the validity of the analyses. From the comparison of the theoretical results and Monte-Carlo simulations, we see that the analytic results match the simulations well. It can also be seen that, although we made some

[‡]As described in Sec III-B, SIMO can be considered as a special multi-antenna transmission mode.

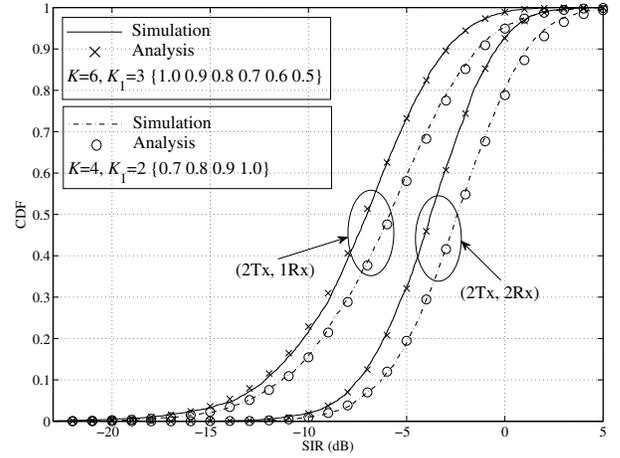


Fig. 1. Impact of total equivalent interfering signal power, $\sum_{k=1}^K P_k\mu_k$ (Both category MIMO modes have distinct power/path-loss)

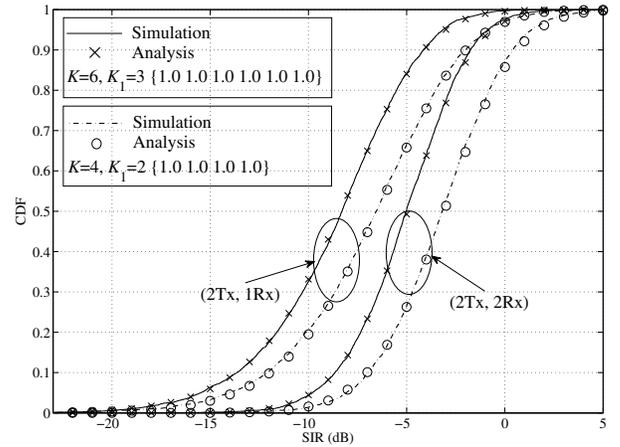


Fig. 2. Impact of total equivalent interfering signal power, $\sum_{k=1}^K P_k\mu_k$ (Both category MIMO modes have equal power/path-loss)

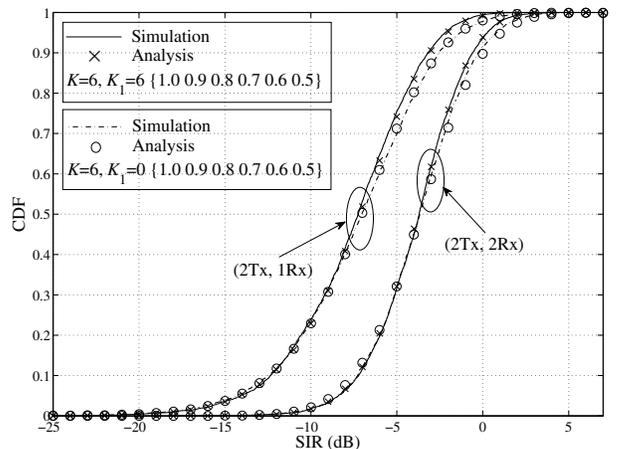


Fig. 3. Impact of different category MIMO modes (Same category MIMO modes have distinct power/path-loss)

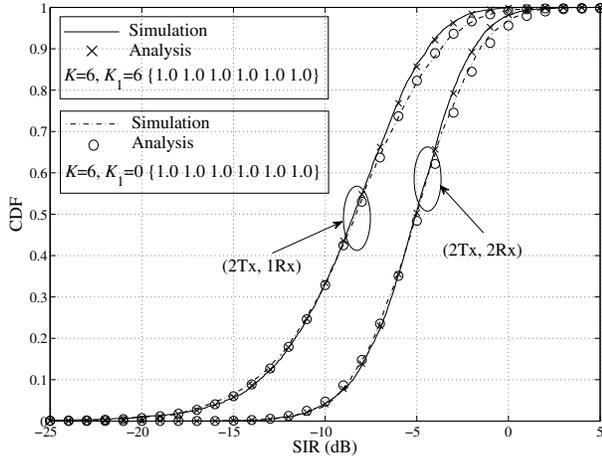


Fig. 4. Impact of different category MIMO modes (Same category MIMO modes have equal power/path-loss)

approximations in deriving the SIR expressions for Category II interfering MIMO modes, the error is relatively small. Moreover, due to the diversity gain from using MRC in the desired STBC receiver, the performance improves with an increase in the number of receiving antennas.

In Figs. 1 and 2, we also investigate the impact of the total equivalent interfering signal power (proportional to $\sum_{k=1}^K P_k \mu_k$). Because the total equivalent interfering signal power for $K = 6$ is set larger than that for $K = 4$, as expected, the performance is better for the latter case.

Figs. 3 and 4 compare the impact of the two categories of MIMO modes on the desired receiver when the total equivalent interfering signal power is fixed. Besides the improvement from an increase in the number of receive antennas, another observation is that the curves for the two categories under the same conditions (same M and $\sum_{k=1}^K P_k \mu_k$) have a crossover at some SIR value. Beyond this crossover, the Category II MIMO schemes have less impact on the desired link. Below this crossover (corresponding to the low-SIR region), the two categories have almost the same impact on the desired link. For the cell-edge user, the low-SIR region is of interest. In this region, the gap between the two categories is small. So, we can use the simplest co-channel scheme (SIMO) to simplify the analysis. For the high-SIR region, we can choose the worst-case scheme of Category I (such as STBC) to design the system. This design is robust to uncertainties about the MIMO schemes in the interfering links.

Another focus in this section is the performance gap between the two categories. From (8), we know that the mean and variance of the co-channel interference term for Category I are $\sum_{k=1}^K P_k \mu_k$ and $\sum_{k=1}^K P_k \mu_k$, respectively. For Category II interferers, the parameters are $\sum_{k=1}^K P_k \mu_k$ and $2 \sum_{k=1}^K P_k \mu_k$. The performance gap between the two categories arises from the difference in the variances. Hence, when $\sum_{k=1}^K P_k \mu_k$ is fixed (see Figs. 3-4), a change in the number of receive

antennas does not have a significant effect on the gap.

VI. CONCLUSION

With a realistic channel model including the effects of path-loss and Rayleigh fading, the statistical distributions of the post-processing SIRs for Alamouti Coding with arbitrary MIMO modes for interfering BSs are derived, and verified by Monte-Carlo simulations. The results show that the two categories have different impact on the desired Alamouti Coding receiver. This research has a reference value in the design of dynamic resource allocation algorithms when MIMO mode adaptation is permitted.

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