Performance Analysis of Optimal Joint Beamforming in Multi-Keyhole MIMO Channels

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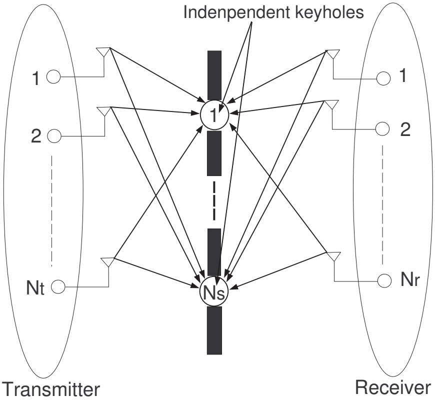
Outline

1. Background
   - Multi-keyhole MIMO Channel Model
   - Joint Transmit and Receive Optimal Beamforming System
2. Statistics of the Maximum Eigenvalue $\lambda_{\max}$ of $\mathbf{H}^{\dagger}\mathbf{H}$
   - Exact c.d.f. and p.d.f. expressions
   - Asymptotic c.d.f. and p.d.f. expressions
3. Outage Performance Analysis
   - Outage probability expressions
   - Impact of keyhole power distribution
4. Conclusion

Multi-keyhole MIMO channels



- $\mathbf{H} = \mathbf{H}_r \mathbf{A} \mathbf{H_t}^{\dagger}$
- N_t transmit antennas, N_s independent keyholes, N_r receive antennas

- Exploits instantaneous channel knowledge to maximize received SNR.
- Transmit BF vector:

$$\mathbf{w}_t = \max \mathsf{vec}(\mathbf{H}^{\dagger}\mathbf{H}) \tag{1}$$

- Receive MRC vector:

$$\mathbf{w}_r = \mathbf{w}_t \mathbf{H}^{\dagger} \tag{2}$$

- Equivalent SISO eigenmode transmission:

$$y = \sqrt{\lambda_{\max}} x + n, \tag{3}$$

  where $\lambda_{\max}$ is the maximum eigenvalue of $\mathbf{H}^{\dagger}\mathbf{H}$
- Output SNR statistics directly determined by maximum eigenvalue statistics of $\mathbf{H}^{\dagger}\mathbf{H}$

Key steps in deriving the c.d.f. of the maximum eigenvalue of $\lambda_{\max}(\mathbf{H}_t\mathbf{A}^\dagger\mathbf{H}_r^\dagger\mathbf{H}_r\mathbf{A}\mathbf{H}_t^\dagger)$:

- Consider case: $\min(N_t, N_r) \geq N_k$
- C.d.f. expression for $\mathbf{H}_t\mathbf{W}\mathbf{H}_t^\dagger$ conditioned on $\mathbf{W}$, $\mathbf{W} \triangleq \mathbf{A}^\dagger\mathbf{H}_r^\dagger\mathbf{H}_r\mathbf{A}$
- Average over the joint p.d.f. of N_k eigenvalues of random matrix $\mathbf{W}$
- Similar analysis for $\min(N_t, N_r) < N_k$
- Unifying expression

Exact c.d.f. expression

Theorem 1

The c.d.f. of the maximum eigenvalue of $\mathbf{F}$ is given by

$$F_{\lambda_{\max}}(x) = \frac{(-1)^{\frac{p(p-1)}{2}} \det(\mathbf{\Phi}(x))}{\prod_{i=1}^{p} \Gamma(n-i+1) \prod_{i<j}^{N_k} (b_j - b_i)}, \qquad (4)$$

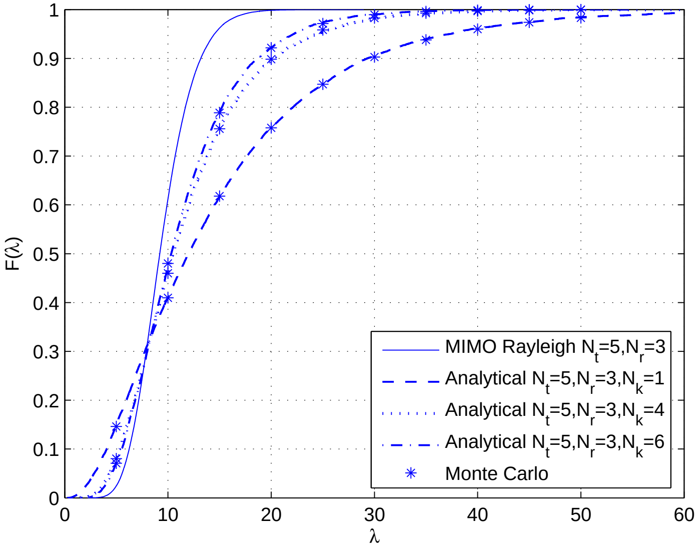
where $\mathbf{\Phi}(x)$ is an $N_k \times N_k$ matrix whose (l,k)th entry is given by

$$[\mathbf{\Phi}(x)]_{l,k} = \begin{cases} b_l^{k-1} & \text{if } k \le N_k - p, \\ g(x)_{l,k} & \text{if } k > N_k - p, \end{cases} \qquad (5)$$

where $g(x)_{l,k}$ is given by

$$g(x)_{l,k} = \Gamma(q-k+1) b_l^{2N_k-p-k} - b_l^{N_k-n-1} \sum_{t=0}^{m+q-n-k} \frac{x^t}{\Gamma(t+1)} 2(b_l x)^{\frac{q-t-k+1}{2}} K_{q-t-k+1}\left(2\sqrt{\frac{x}{b_l}}\right). \qquad (6)$$

$m = \max(N_t, N_r)$, $n = \min(N_t, N_r)$, $q = \max(n, N_k)$, $p = \min(n, N_k)$



1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
F(λ)
0
10
20
30
40
50
60
λ
MIMO Rayleigh N_t=5,N_r=3
Analytical N_t=5,N_r=3,N_k=1
Analytical N_t=5,N_r=3,N_k=4
Analytical N_t=5,N_r=3,N_k=6
Monte Carlo

Theorem 2

When $n = 1$, the asymptotic expansions of the c.d.f. of the maximum eigenvalue $\lambda_{\max}$ of $\mathbf{F}$ are given by

$$F_{\lambda_{\max}}(x) = \frac{a_1}{d} x^d + o(x^d), \tag{7}$$

where $d = \min(m, N_k)$, and

$$a_1 = \begin{cases} \frac{\Gamma(m-N_k)}{\Gamma(m)\Gamma(N_k)\prod_{i=1}^{N_k} b_i} & \text{if } m > N_k, \\ \frac{1}{\Gamma(m)^2}\left(\frac{\psi(1)+\psi(m)-\ln x}{\prod_{i=1}^{N_k} b_i} + \frac{(-1)^{m-1}\det(\mathbf{\Phi}^3)}{\prod_{i<j}^{N_k}(b_j-b_i)}\right) & \text{if } m = N_k, \\ \frac{(-1)^{m-1}}{\Gamma(m)^2}\frac{\det(\mathbf{\Phi}^4)}{\prod_{i<j}^{N_k}(b_j-b_i)} & \text{if } m < N_k, \end{cases} \tag{8}$$

where $\mathbf{\Phi}^3$ and $\mathbf{\Phi}^4$ are $N_k \times N_k$ matrices with entries

$$[\mathbf{\Phi}^3]_{l,k} = \begin{cases} b_l^{k-1} & \text{if } k = 1, \cdots, N_k - 1, \\ b_l^{-1}\ln b_l & \text{if } k = N_k, \end{cases} \tag{9}$$

and

$$[\mathbf{\Phi}^4]_{l,k} = \begin{cases} b_l^{k-1} & \text{if } k = 1, \cdots, N_k - 1, \\ b_l^{N_k-m-1}\ln b_l & \text{if } k = N_k, \end{cases} \tag{10}$$

respectively.

Outage probability expression

Corollary 1

The outage probability of the optimal MIMO beamforming system in multi-keyhole channels can be expressed as

$$\mathrm{P_{out}}(\gamma_{\mathrm{th}}) = \frac{(-1)^{\frac{p(p-1)}{2}} \det\left(\mathbf{\Phi}\left(\frac{\gamma_{\mathrm{th}}}{\gamma}\right)\right)}{\prod_{i=1}^{p}\Gamma(n-i+1)\prod_{i<j}^{N_k}(b_j-b_i)}, \tag{11}$$

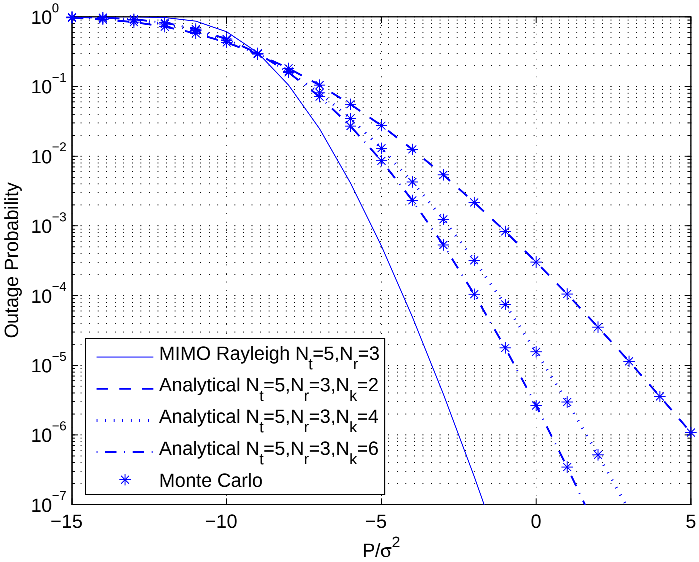
where $\mathbf{\Phi}(x)$ has been defined in (5).

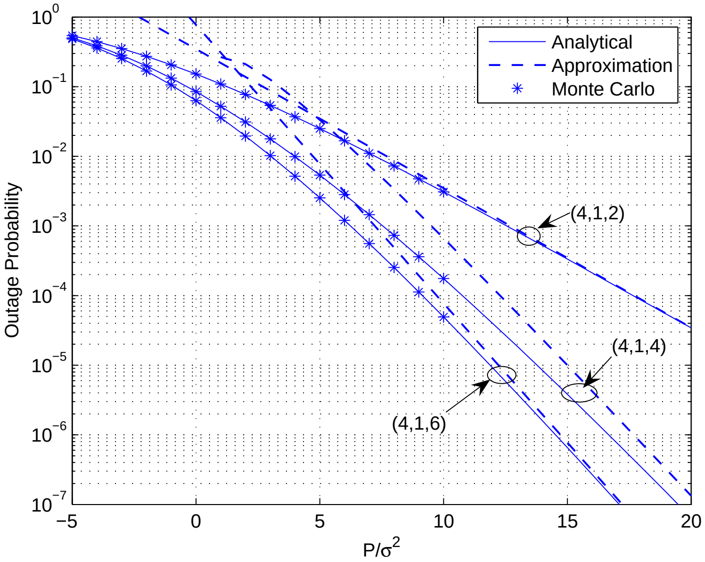
Corollary 2

At high SNR regime, the outage probability of the optimal beamforming system in MISO/SIMO multi-keyhole channels can be approximated as

$$\mathrm{P_{out}^{n=1}}(\gamma_{\mathrm{th}}) \approx \begin{cases} \frac{\Gamma(m-N_k)}{\Gamma(m)\Gamma(N_k+1)\prod_{i=1}^{N_k} b_i}\left(\frac{\gamma_{\mathrm{th}}}{\gamma}\right)^{N_k} & \text{if } m > N_k, \\ \frac{1}{\Gamma(m)\Gamma(m+1)\prod_{i=1}^{N_k} b_i}\ln\left(\frac{\gamma}{\gamma_{\mathrm{th}}}\right)\left(\frac{\gamma_{\mathrm{th}}}{\gamma}\right)^{N_k} & \text{if } m = N_k, \\ \frac{(-1)^{m-1}}{\Gamma(m)\Gamma(m+1)}\frac{\det(\mathbf{\Phi}^4)}{\prod_{i<j}^{N_k}(b_j-b_i)}\left(\frac{\gamma_{\mathrm{th}}}{\gamma}\right)^{m} & \text{if } m < N_k. \end{cases} \tag{12}$$

Numerical result-Exact Outage Probability





Outage Probability
P/σ²
Analytical
Approximation
Monte Carlo
(4,1,2)
(4,1,4)
(4,1,6)

Corollary 2

When $m \geq N_k$, $\mathrm{P}_{\mathrm{out}}^{n=1}(\gamma_{\mathrm{th}})$ is Schur-convex function with respect to b_i, $i = 1, \cdots, N_k$.

Example

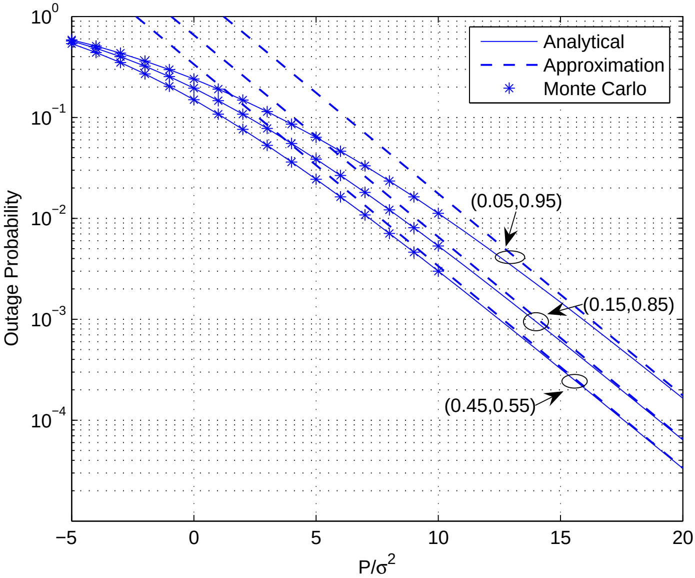
$N_t = 4$, $N_r = 1$, $N_k = 2$, consider three cases with keyhole power distribution $(0.05, 0.95)$, $(0.15, 0.85)$ and $(0.45, 0.55)$, respectively. Since the power distribution has the following majorization relationship

$$(0.05, 0.95) \succ (0.15, 0.85) \succ (0.45, 0.55), \tag{13}$$

Corollary 2 indicates that

$$\mathrm{P}_{\mathrm{out}}^{n=1}(\gamma_{\mathrm{th}})_{(0.05,\ 0.95)} \geq \mathrm{P}_{\mathrm{out}}^{n=1}(\gamma_{\mathrm{th}})_{(0.15,\ 0.85)} \geq \mathrm{P}_{\mathrm{out}}^{n=1}(\gamma_{\mathrm{th}})_{(0.45,\ 0.55)}. \tag{14}$$

Numerical results



Conclusion

- Derived closed-form and asymptotic expressions for the c.d.f. of maximum eigenvalue of the product of two independent complex Gaussian matrices with correlation
- Analyzed the outage performance of transmit beamforming and receive combining systems in Multi-keyhole MIMO channels
- Characterized the impact of power distributions among keyholes on the outage performance