Cyclostationary Signatures in OFDM-Based Cognitive Radios With Cyclic Delay Diversity

Shanghai Research Center for Wireless Communications (WiCO), China

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Outline

1 Introduction

2 System Model

3 Intrinsic Cyclostationary Signatures in CDD-OFDM

4 Application to Spectrum Sensing
   - Asymptotical CFAR Testing Based on Multiple Lags
   - Numerical Results

5 Conclusions
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Introduction

Cyclostationary signature for cognitive radios

As defined in [4], a cyclostationary signature is a feature, intentionally embedded in the physical properties of digital communications signal, which may be easily generated, manipulated, detected and analyzed using low-complexity transceiver architecture.

The cyclostationary signatures provide a robust mechanism for signal detection, network identification and signal acquisition as part of the process of network coordination without the requirement of a dedicated control channel.
### Cyclostationarity of OFDM Signals

<table>
<thead>
<tr>
<th>CP-induced cyclostationarity</th>
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System Model: CDD-OFDM

Transmit architecture of OFDM system utilizing CDD

\[ \{c_{1,k}\}_{k=0}^{N-1} \rightarrow \text{OFDM Modulation} \rightarrow \tilde{s}_1[k] \rightarrow \frac{1}{\sqrt{N_T}} \rightarrow \Delta_2 \rightarrow \text{Cyclic Prefix} s_{l,1}[k] \yup \rightarrow \Delta_n \rightarrow \text{Cyclic Prefix} s_{l,2}[k] \yup \rightarrow \Delta_{N_T} \rightarrow \text{Cyclic Prefix} s_{l,nr}[k] \yup \rightarrow \text{Cyclic Prefix} s_{l,N_T}[k] \yup \]
Standard conformability

- Implement only in relays being transparent to destination receiver side
- It can be incorporated within the OFDM-based standards such as WiMAX, 3GPP-LTE, and IEEE 802.11a etc., considering the size and cost of multiple antennas is prohibitive for wireless devices.
CDD-OFDM: Appealing Features

Standard conformability

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CDD-OFDM: Appealing Features

1. Standard conformability
2. Delay diversity gain

- **Delay diversity gain**
  - Convert virtual MISO channel into an equivalent SISO channel with increased frequency diversity.
  - Transform delay diversity into frequency diversity
  - Collect increased diversity by an outer error control coding such as convolutional coding
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CDD-OFDM: Appealing Features

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3. Saturation effect

Saturation effect [1]

\[ \Delta n_T \geq \frac{1}{B T_s} \quad (n_T = 1, 2, \ldots, N_T) \] is a saturation region in terms of cyclic delays, where \( B \) is bandwidth of OFDM signal and \( T_s \) is sample period.

- In the saturation region, the system can achieve almost the same delay diversity gain approaching to the maximum.
- Saturation effect allows for tuning cyclic delays for other metrics, while keeping the desirable performance of antenna system.
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System Model: Signal Formulation

\[ s_{l,n_T}(k) = \frac{1}{\sqrt{N_T}} \tilde{s}_l[(k - \Delta_{n_T}) \mod N], \quad (1) \]

where \( \Delta_{n_T} \) is the antenna-dependent cyclic delay (\( 0 = \Delta_1 < \Delta_2 < \cdots < \Delta_{N_T} \)).

\[ s_{n_T}(n) = \frac{1}{N_T} \sum_{l=-\infty}^{+\infty} g(n - lM) \sum_{k=0}^{N-1} c_{l,k} W_N^{k\Delta_{n_T}} W_N^{k(lM-n)} \quad (2) \]

where \( M = N + N_G \) and \( g(n) = R_{[0,M-1]}^{(n)} \) with

\[ R_{[T_1,T_2]}^{(n)} = \begin{cases} 1 & n = T_1, T_1 + 1, \cdots, T_2 \\ 0 & \text{else} \end{cases} \quad (3) \]

The CDD-OFDM signal received by one antenna can be written as

\[ r(n) = \sum_{l=0}^{L_h} h_l s(n - l) + w(n) \quad (4) \]
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Cyclostationary Characteristics of CDD-OFDM Signal

- Defining the correlation matrix of the vector random process $\mathbf{s}(n)$ as

$$
\mathbf{C}_s(n, \tau) = E\{\mathbf{s}(n)\mathbf{s}^H(n + \tau)\}
$$

(5)

- Cyclic Autocorrelation Function (CAF)

$$
\tilde{c}_r(k, \tau) = \sum_{n=0}^{M-1} c_r(n, \tau) W_M^{kn}
$$

(6)

$$
= \sum_{l=0}^{L_h} h_l W_M^{kl} \sum_{r=\tau+l-L_h}^{\tau+l} \tilde{C}_s(k, r) h_{\tau+l-r}^H + c_w(\tau) \delta(k)
$$
Cyclostationary Characteristics of CDD-OFDM Signal

\[ [C_s(n, \tau)]_{i,j} = \frac{1}{N_T} \sum_{l=-\infty}^{+\infty} g(n - lM)g(n + \tau - lM)\delta_N[\tau - (\Delta_j - \Delta_i)] \]

\[
= \begin{cases} 
\frac{1}{N_T} \sum_{l=-\infty}^{+\infty} R^{(n-lM)}_{[0,M-1-(\Delta_j-\Delta_i)]} & 0 \leq \tau = (\Delta_j - \Delta_i) \\
\frac{1}{N_T} \sum_{l=-\infty}^{+\infty} R^{(n-lM)}_{[-(\Delta_j-\Delta_i),M-1]} & \tau = (\Delta_j - \Delta_i) < 0 \\
\frac{1}{N_T} \sum_{l=-\infty}^{+\infty} R^{(n-lM)}_{[0,M-1-N-(\Delta_j-\Delta_i)]} & 0 \leq \tau = N + (\Delta_j - \Delta_i) \leq M-1 \\
\frac{1}{N_T} \sum_{l=-\infty}^{+\infty} R^{(n-lM)}_{[N-(\Delta_j-\Delta_i),M-1]} & 1 - M \leq \tau = -N + (\Delta_j - \Delta_i) \leq 0 \\
0 & \text{else}
\end{cases}
\]
Intrinsic Cyclostationary Signatures in CDD-OFDM

\( N_T = 2, N = 32, N_G = 8 \)

\[ \Delta_2 = 4 \]

\[ \Delta_2 = 10 \]

A sequence of lag-indexed spectrum lines indexed by the following indices set

\[ \Omega = \{ \tau | \tau = \pm (\Delta_j - \Delta_i), N \pm (\Delta_j - \Delta_i), -N \pm (\Delta_j - \Delta_i); i, j = 1, 2, \ldots, N_T \}. \] (8)
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Asymptotical CFAR testing based on multiple lags

The hypothesis testing for the presence of primary user can transform to the problem for testing if \( \alpha \) is a cyclic frequency, formulated as

\[
H_0 : \forall \tau \Rightarrow \hat{c}_r(\alpha, \tau) = \varepsilon(\alpha, \tau);
\]
\[
H_1 : \text{For some } \tau \subseteq \Omega \Rightarrow \hat{c}_r(\alpha, \tau) = c_r(\alpha, \tau) + \varepsilon(\alpha, \tau)
\]

\( \varepsilon(\alpha, \tau) : \lim_{L \to \infty} \sqrt{L} \varepsilon(\alpha, \tau) \sim \mathcal{N}(0, \Sigma_r(\alpha, \tau)) \).

Generalized Likelihood Ratio (GLR):

\[
T_r(\alpha, \tau) = -2 \ln \Lambda = L\hat{c}_r(\alpha, \tau)\hat{\Sigma}_r^{-1}(\alpha, \tau)\hat{c}_r^T(\alpha, \tau).
\]

Under the null hypothesis, \( T_r(\alpha, \tau) \) is asymptotically \( \chi^2_{2N_\tau} \) distributed. As a result, we can present the test which is based on a CFAR approach for selecting a threshold.
Common Simulation Parameters

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>DESCRIPTION</th>
<th>VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>DFT size</td>
<td>128</td>
</tr>
<tr>
<td>$\Delta_f$</td>
<td>Subcarrier frequency spacing</td>
<td>10.9325 kHz</td>
</tr>
<tr>
<td>$f_s$</td>
<td>Sampling frequency</td>
<td>2.798720 MHz</td>
</tr>
<tr>
<td>$N_G/N$</td>
<td>CP ratio</td>
<td>1/8</td>
</tr>
<tr>
<td>$f_c$</td>
<td>Carrier frequency</td>
<td>2.5 GHz</td>
</tr>
<tr>
<td>$T_o = 1/\Delta_f$</td>
<td>OFDM symbol duration without CP</td>
<td>91.43 $\mu$s</td>
</tr>
<tr>
<td>$T$</td>
<td>OFDM symbol duration with CP</td>
<td>102.86 $\mu$s</td>
</tr>
<tr>
<td>Modulation</td>
<td></td>
<td>16QAM</td>
</tr>
<tr>
<td>$N_T$</td>
<td>Number of transmit antennas</td>
<td>2</td>
</tr>
<tr>
<td>$v$</td>
<td>Velocity</td>
<td>0 m/s</td>
</tr>
<tr>
<td>$SNR$</td>
<td>Signal-to-noise ratio</td>
<td>$-10 \log \sigma_w^2$ dB</td>
</tr>
<tr>
<td>$L$</td>
<td>Length of observations</td>
<td>$10 \times T$</td>
</tr>
<tr>
<td>$L_w$</td>
<td>Length of Kaiser window</td>
<td>1029</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$\beta$ parameter of Kaiser window</td>
<td>10</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Given cyclic frequency for detection</td>
<td>$1/T$</td>
</tr>
</tbody>
</table>
Typical set of multiple lags

From the set of $\Omega$, we adopt the following typical sets as observation spots

$$\tau_1 = [-128, -128 + \Delta_2, -\Delta_2, \Delta_2, 128 - \Delta_2, 128], \quad (\Delta_2 \neq 64)$$

$$\tau_2 = [-128, 128]$$

$$\tau_3 = [-128 - \Delta_2, -128, -128 + \Delta_2, -\Delta_2, \Delta_2, 128 - \Delta_2, 128, 128 + \Delta_2], \quad (\Delta_2 \leq 15)$$

$$\tau_4 = [-128, -64, 64, 128].$$

Detection probability versus cyclic delay with different methods

$SNR = -12$ dB

$PFA = 0.05$
Simulation results

Detection probability versus SNR with different methods

Receiver operating characteristics with different methods

Detection probability versus SNR with different methods:
- \( \tau_1 \)-detector, \( \Delta_2 = 16 \)
- \( \tau_1 \)-detector, \( \Delta_2 = 61 \)
- \( \tau_2 \)-detector, \( \Delta_2 = 16 \)
- \( \tau_2 \)-detector, \( \Delta_2 = 61 \)

PFA = 0.05

SNR = -12 dB

Detection Probability

SNR (dB)

Detection Probability

PFA

0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0

0.0 0.2 0.4 0.6 0.8 1.0
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The procedure of Cyclic Delay Diversity (CDD) simultaneously possess the dual advantages in terms of antenna diversity and cognitive radios.

- For the OFDM-based cognitive radios, the CDD procedure can be characterized as a cost-efficient approach to generating flexible cyclostationary signatures.

- The CDD-induced cyclostationary signatures may be easily implemented, manipulated, detected and analyzed using the standard compatible CDD-OFDM architectures without suffering signaling overhead.

- The novel approach still achieves the initial goal toward the delay diversity gain which originally drives the CDD procedure into real applications.